The Search Costs of Inflation*

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Abstract

What are the costs of inflation in the labor market? When wages are set nominally, inflation leads to reduced purchasing power, which prompts workers to search for other jobs in order to adjust their real wages; this search is costly. We quantify these costs in a model of on-the-job search, where search effort responds endogenously to expected and unexpected inflation. Inflation erodes the value of a match to a worker through real wage losses and larger incurred search costs. The real wage loss is offset as a benefit to firms, whereas the cost of search is a net aggregate cost of inflation.

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1 Introduction

Recent work finds that, in the presence of nominally rigid wages, workers respond to elevated inflation expectations via on-the-job search (Pilossoph and Ryngaert (2024)). Allowing them to respond to realized inflation in the same way implies a positive correlation between underlying inflation, search effort, and the employer-to-employer transition rate. This suggests a new "cost" of inflation not captured by previous theories, the search cost, that may be partially offset by the productive reallocation of workers. This paper outlines a new theory of and attempts to quantify these search costs of inflation.

In order to consider the welfare consequences of inflationary shocks, we extend the canonical model of Postel-Vinay and Robin (2002) to include nominal wage contracting, endogenous search effort, and a price level featuring deterministic trend inflation with shocks around its trend. Wage contracts adjust with trend inflation, akin to a cost-of-living (COLA) adjustment. However, inflationary shocks around this trend have both redistributive and allocative consequences. Due to a two-sided lack of commitment, nominal wages of incumbent workers only adjust through credible outside offers. Therefore, inflationary shocks reduce real wages for incumbent workers - transferring this surplus to employers as profits. As a result, workers choose optimal search effort to adjust the likelihood of receiving an outside offer, to either change jobs or renegotiate their current wage. That is, the reduction in real wages induces them to increase their search effort at a cost.

This has two key implications. First, since the search costs incurred by workers are a form of deadweight loss, unexpected inflation incurs aggregate welfare losses above and beyond the redistribution of welfare from workers to firms. Second, and as a direct consequence of the rise in search effort, inflation shocks have allocative consequences. As search effort increases, workers may adjust their wages upward while remaining with their employer, but they may also move up the job ladder to more productive firms. Together, these mechanisms determine the aggregate welfare effects of unexpected inflationary shocks, which may have a net positive or net negative effect.

These mechanisms follow from the structure of wage determination in our model. A nominal wage and its growth rate are contracted upon at the start of the match. The terms are fixed for the duration of the match unless either party has a credible threat which induces the nominal wage to change. This results in sticky nominal wages which imply real wages movements in response to unanticipated movements in the price level. Nominal wages are re-negotiated in a new contract between an employer and employee when (i) the worker receives an outside offer resulting in a job-to-job transition, (ii) the worker obtains an outside offer resulting in a renegotiation of terms with their current employer, or (iii) inflation induces a large enough reduction (increase) in the worker's purchasing power that the worker's (firm's) net value of the match becomes negative. Nominal wage rigidity therefore results in workers facing real wage cuts absent an outside offer, which prompts them to increase their search effort. This increases the likelihood of job-to-job transitions, but also generates a deadweight loss by increasing search costs that are incurred.

We calibrate our model to match key features of the pre-pandemic US labor market, including the real wage growth of stayers and movers, the search-wage elasticity, and the offer yield between the employed and non-employed. Using our calibrated model, we first provide a quantitative estimate of the welfare consequences of an unanticipated shock to annual trend inflation of 12 percentage points, so that US trend inflation reaches the world average inflation rate in 2023. While trend inflation itself has no allocative role in our economy, transitions from one trend inflation regime to another result in both worker and aggregate welfare losses. Even two years after the shock, flow worker welfare is 7% lower relative to the economy that experiences no shock, and aggregate welfare is 1% lower. The loss in worker welfare is a combination of real wage losses - which are a net positive to the firm - and search costs, which are a net loss in the economy. In a second exercise, we provide a quantitative estimate for the welfare consequences of an unanticipated, temporary shock to inflation around its trend whose magnitude is calibrated to the COVID-era period. Lifetime welfare losses experienced by agents in the shocked economy amount to 1% percent, suggesting that the reallocative channel of unexpected inflation that moves workers up the productivity ladder is dominated by the deadweight loss induced by increased search effort.

The costs that we estimate qualitatively align with both historical and recent evidence documenting how workers *feel* about inflation. In earlier work, Shiller (1997) surveyed households on why they dislike inflation, and found that the key reason was their declining real incomes. More recently, Stancheva (2024) revisits the same question using new survey methods, and similarly finds that if there is one main reason workers dislike inflation, "it is because many individuals feel that it systematically erodes their purchasing power," a finding similar to that of Hajdini et al. (2022). [1] Our framework takes these observations seriously, and additionally considers the costly actions workers undertake to combat these real wage declines.

To this end, our paper is most closely related to contemporaneous work by Afrouzi et al. (2024b) and Guerreiro et al. (2024). Both of these papers offer theories of why workers dislike inflation, part of which is real wage losses, as in our framework. Afrouzi et al. (2024b) features search effort costs like ours, but also introduces renegotiation costs, the central cost in Guerreiro et al. (2024). While renegotiation in our framework is not costly per se, it is more likely to happen once costly search effort is undertaken, so there is a tight link between search costs and the effective costs of renegotiation.

Additionally, relative to these papers our framework features a job ladder with heterogeneous

[1] Guerreiro et al. (2024) also provide new survey evidence suggesting that workers "are willing to sacrifice 1.75% of their wages to avoid conflict associated with bargaining with employers. Afrouzi et al. (2024a) also provide survey evidence documenting that people prefer lower inflation to higher inflation.

firm productivity, though we abstract from worker heterogeneity. This allows us quantify how much wage growth after costly search comes with productivity growth. Moscarini and Postel-Vinay (2022) argue that job-to-job transitions result from the reallocation of employees from less-productive jobs to more-productive jobs. These types of wage increases are not inflationary in their environment because firms realize productivity gains that offset the higher wages they pay. On the other hand, offers that prompt renegotiation at the current firm are inflationary, as they act as marginal cost shocks. While the pricing side of our framework is exogenous and therefore far less rich than Moscarini and Postel-Vinay (2022), on the labor market side we allow for the endogenous response of the contact rate to inflation via search effort, showing that inflationary shocks will increase both job-to-job transitions and counteroffers simultaneously. This suggests a potential mechanism for so-called wage-price spirals in which prices and wages increase in response to one another (Blanchard (1986)).[2] For example, Faccini and Melosi (2023) and Karahan et al. (2017) show that the on-the-job search rate and the rate of job-to-job transitions predict future inflation, so a wage-price spiral seems plausible once search effort itself responds to inflation.

More broadly, this paper contributes to the literature on estimating and explaining the passthrough of price inflation to wage inflation. Hajdini et al. (2022), Jain et al. (2022), and Buchheim et al. (2023) find evidence that the perceived passthrough of inflation to wage growth is low. Pilossoph and Ryngaert (2023) provide evidence that this prompts employed workers to search and potentially speed the arrival of negotiations. Buchheim et al. (2023) show that - among German workers and firms - expected passthrough increases when workers and firms anticipate negotiations to take place. Higher inflation expectations do not, however, increase the likelihood that German workers ascribe to entering into negotiations. The current paper combines nominal wage rigidity and endogenous search effort to evaluate the mechanisms of passthrough of realized inflation to wage growth.

2 Model

We now outline a model of search on- and off-the-job in an economy with exogenous aggregate productivity z which is growing deterministically at rate g_z. The exogenous price level in the economy consists of two components: a deterministic trend component p and a stochastic component p_ε. The deterministic trend component grows at rate g_p:

$$p' = p\,(1 + g_p)$$

[2]Faccini and Melosi (2023) allow the arrival rate of offers to change with the rate of on-the-job search and show that an increase in the rate of job-to-job transitions is theoretically consistent with wage pressure as it provides a measure of the competition between firms for workers

The stochastic price level component grows at rate ε_p:

$$p'_\varepsilon = p_\varepsilon \left(1 + \varepsilon'_p\right)$$

where the shock ε'_p follows:

$$\varepsilon'_p = \rho \varepsilon_p + \nu', \quad \nu' \sim N\left(0, \sigma^2_\nu\right)$$

There is a unit mass of firms, each with a vacancy, indexed by their productivity $y \in \left(\underline{y}, \bar{y}\right)$. The exogenous distribution of vacancies across firms y is denoted by $F(y)$.[3] Per-period real output between a worker and a firm of type y is given by $Y(z, y) = zy$.

Workers are homogeneous, infinitely lived, and of measure one, with linear preferences over a single final consumption good. They can either be employed or unemployed, and we denote those states by $i \in \{e, u\}$, respectively. Both employed and unemployed workers make search effort decisions $s \in (0, \bar{s}_i)$, which determine the rate at which they meet available vacancies. For employed workers, the rate is $\lambda_e + s$ and for the unemployed it is $\lambda_u + s$. The real cost of search is given by $C(z, s)$, which is increasing and convex in search effort, $\frac{\partial C(z,s)}{\partial s} > 0$, $\frac{\partial^2 C(z,s)}{\partial^2 s} > 0$. Workers exogenously separate at rate δ from their jobs, and earn a real flow value of unemployment $B(z)$. All agents discount the future at rate β.

2.1 Wage Setting and Wage Contracts

When a worker and a firm meet, they decide on an initial nominal wage w, and agree that it will grow at rate $(1 + g_z)(1 + g_p)$ following the deterministic growth in the economy absent any outside events which may trigger a change.[4]

What kind of events can change the path of wages? The wage level will be renegotiated only by mutual consent; this can happen if a worker receives an outside offer which dominates what they currently make, forcing the firm to adjust the base pay upward, if it is feasible. In this case, wages are determined by Bertrand competition between the incumbent firm and the poaching firm (Postel-Vinay and Robin (2002)). In some cases, the incumbent firm will not be able to provide a wage which dominates the new offer, and the worker will leave to the new firm. The wage they receive there will also be determined by Bertrand competition between the incumbent firm and the poaching firm. Alternatively, a shock to the stochastic price level component ε_p may render the real wage too low (high), which will cause the nominal wage to

[3] In the model we outline below, it is theoretically straightforward to introduce endogenous vacancy creation. However, with endogenous search effort, the vacancy creation decision becomes a function of the distribution of workers across firms and the unemployment rate, which substantially increases the computational burden of the model.

[4] Wages are indexed to trend inflation g_p but not to expected inflation, which would depend on the current realization of ε_p when $\rho > 0$. This assumption reflects the common practice of cost-of-living adjustments (COLAs) being tied to known trend inflation rates rather than volatile expectations.

be adjusted upward (downward), as we describe below.

When firms meet unemployed workers, they make take-it-or-leave-it (TIOLI) offers, that is a wage $w = \phi_u$ such that the worker is indifferent between employment and unemployment. The value of unemployment to a worker can thus be written as:

$$\begin{aligned} U(z) &= \max_{s \in [0, \bar{s}_u]} B(z) - C(z, s) + \beta (1 - (s + \lambda_u)) U(z') \\ &\quad + \beta \mathbf{E}_{\varepsilon_p'} (\lambda_u + s) \int_y \max \Big\{ U(z'), W\left(\phi_u, y, p', p_\varepsilon', \varepsilon_p', z'\right) \Big\} dF(y) \\ &= B(z) + \beta U(z') \end{aligned}$$

The second equality comes as a direct result of the TIOLI offers assumption: firms will offer workers a wage that is just large enough so that they are indifferent between taking the offer and remaining unemployed; as such, they receive the same value regardless, so that the future value collapses to a simple expression. Because workers get the same value whether they search and search is costly, all unemployed workers choose $s_u^* = 0$.[5]

Turning to the employed, consider a worker currently with a firm of type y_1 making a nominal wage w that is contracted to grow at a rate $(1 + g_z)(1 + g_p)$. Absent any contact with another firm, the worker's wage may adjust if the realized real wage is either too high or too low at the future realized price level. We characterize this adjustment momentarily, but for now assume that their baseline wage will be w' after this adjustment, and now assume the worker is contacted by a firm y_2. Bertrand competition between employers will lead to one of the three following outcomes:

Case 1: No outbidding, contract remains the same. The worker will not be able to use the outside offer to change their current contract if $y_2 < y_1$ and

$$W\Big(z'p'p_\varepsilon' y_2, y_2, p', p_\varepsilon', \varepsilon_p', z'\Big) \leq W\Big(w', y_1, p', p_\varepsilon', \varepsilon_p', z'\Big)$$

where $w' = w(1 + g_z)(1 + g_p)$. Why? Because the maximum nominal wage the poaching firm can offer to start is $z'p'p_\varepsilon' y_2$, that is the total output in nominal terms that the pair will produce next period. Since this provides a value to the worker which is lower than the value they receive by remaining at the current firm where their wage will grow to w', the worker has no credible threat, and their contract remains unchanged.

[5] While this is not an accurate description of the search behavior of unemployed workers, our focus is on the search behavior of the employed, so we make this simplifying assumption. The assumption can easily be relaxed by either allowing workers to receive some value in employment that is a share of the value of unemployment (Cahuc et al. (2006)), or by making the value of unemployment depend on the price level through a benefit which is given nominally. In both cases, searching would have a positive return for the unemployed.

Case 2: No outbidding, wage contract is renegotiated to ϕ^{reneg}. Suppose that $y_2 < y_1$, but instead:

$$W\left(z'p'p'_{\varepsilon}y_2, y_2, p', p'_{\varepsilon}, \varepsilon'_p, z'\right) > W\left(w', y_1, p', p'_{\varepsilon}, \varepsilon'_p, z'\right)$$

In this case, the worker indeed has a credible threat since the poaching firm is able to offer $z'p'p'_{\varepsilon}y_2$ and make the worker better off. The incumbent firm then must offer a wage, call it ϕ^{reneg}, which will make the worker indifferent between staying and leaving:

$$W\left(z'p'p'_{\varepsilon}y_2, y_2, p', p'_{\varepsilon}, \varepsilon'_p, z'\right) = W\left(\phi^{\text{reneg}}, y_1, p', p'_{\varepsilon}, \varepsilon'_p, z'\right)$$

Case 3: Poaching firm hires worker at wage ϕ^{poach}. If $y_2 > y_1$, the worker moves to the poaching firm, and gets a new wage ϕ^{poach} which satisfies:

$$W\left(\phi^{\text{poach}}, y_2, p', p'_{\varepsilon}, \varepsilon'_p, z'\right) = W\left(z'p'p'_{\varepsilon}y_1, y_1, p', p'_{\varepsilon}, \varepsilon'_p, z'\right)$$

The above cases imply that there is a cutoff firm, call it $q\left(w', y_1, p', p'_{\varepsilon}, \varepsilon'_p, z'\right)$, which is the lowest productivity firm that triggers a wage renegotiation or a job-to-job transition for a worker at firm y_1 employed at wage w'. This firm is defined implicitly as the firm q such that:

$$W\left(w', y_1, p', p'_{\varepsilon}, \varepsilon'_p, z'\right) = W\left(z'p'p'_{\varepsilon}q, q, p', p'_{\varepsilon}, \varepsilon'_p, z'\right) \tag{1}$$

As we show graphically in Figure 1, if a worker receives an offer from firm $y_2 < q\left(w', y_1, p', p'_{\varepsilon}, \varepsilon'_p, z'\right)$, nothing happens. Above this value, but below the current firm y_1, a wage renegotiation is triggered without a job-to-job transition. Above this value, the worker moves to the new firm.

Importantly, this cut-off firm depends not only on the firm at which the worker is employed, but also on their current nominal wage. For a given price level, the lower the current nominal wage, the lower is the cutoff firm, implying that a larger set of firms will induce a wage renegotiation if the worker receives an outside offer. Moreover, for a given nominal wage, a higher stochastic price level component p_{ε} (corresponding to higher realized inflation ε_p) lowers the real wage, which also lowers the cutoff firm $q(\cdot)$, producing the same effect.

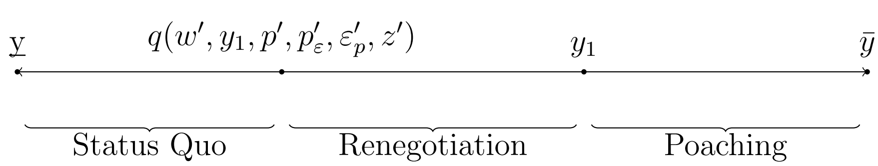


Figure 1: Offers Ranges for Status Quo, Renegotiation, and Poaching.

The $q(\cdot)$ function is central to understanding the effects of shocks to the price level and its trend. For any given benchmark nominal wage w', higher (unanticipated) levels of p' or p'_{ε} lower the workers' real wage, and expand the region of firm offers which trigger renegotiation. This can be seen by inspecting the relationship which implicitly determines $q(\cdot)$ in Equation 1. Since the value of employment will turn out to be decreasing in p and p_ε, all else equal $q(\cdot)$ must fall to satisfy the condition. Moreover, as we show below, a larger range of offers triggering an upward nominal wage change implies that the returns to search rise, so that inflationary shocks raise search effort, increase job-to-job transitions, and also increase the rate of wage re-negotiations conditional on search.

With these cases in mind, we can now write the value of employment to a worker with current nominal wage w employed at firm y when the trend component of the price level is p, the stochastic component is p_ε, the cyclical component is ε_p, and productivity is z below. We will also define w', the benchmark nominal wage used in the previous discussion.

$$
\begin{aligned}
W(w, y, p, p_\varepsilon, \varepsilon_p, z) = & \max_{s\in[0,\bar{s}_e]} \frac{w}{p \cdot p_\varepsilon} - C(s, z) + \beta(\delta + (1-\delta)\lambda_g) U(z') \\
& + \beta \mathbf{E}_{\varepsilon_p'} (1-\delta)(s+\lambda_e) \int_{\underline{y}}^{q\left(w',y,p',p_\varepsilon',\varepsilon_p',z'\right)} W\left(w', y, p', p_\varepsilon', \varepsilon_p', z'\right) dF(x) \\
& + \beta \mathbf{E}_{\varepsilon_p'} (1-\delta)(s+\lambda_e) \int_{q\left(w',y,p',p_\varepsilon',\varepsilon_p',z'\right)}^{y} W\left(\phi^{\text{reneg}}\left(y, x, p', p_\varepsilon', \varepsilon_p', z'\right), y, p', p_\varepsilon', \varepsilon_p', z'\right) dF(x) \\
& + \beta \mathbf{E}_{\varepsilon_p'} (1-\delta)(s+\lambda_e) \int_{y}^{\bar{y}} W\left(\phi^{\text{poach}}\left(x, y, p', p_\varepsilon', \varepsilon_p', z'\right), x, p', p_\varepsilon', \varepsilon_p', z'\right) dF(x) \\
& + \beta \mathbf{E}_{\varepsilon_p'} (1-\delta)(1-s-\lambda_e) W\left(w', y, p', p_\varepsilon', \varepsilon_p', z'\right)
\end{aligned}
$$

where

$$p' = p(1+g_p)$$

$$p_\varepsilon' = p_\varepsilon\left(1+\varepsilon_p'\right)$$

$$z' = (1+g_z) z$$

$$
w' = \begin{cases}
\hat{w} : J\left(\hat{w}, y, p', p_\varepsilon', \varepsilon_p', z'\right) = 0, \text{if } J\left(w(1+g_z)(1+g_p), y, p', p_\varepsilon', \varepsilon_p', z'\right) < 0 \\
\hat{w} : W\left(\hat{w}, y, p', p_\varepsilon', \varepsilon_p', z'\right) - U(z') = 0, \text{if } W\left(w(1+g_z)(1+g_p), y, p', p_\varepsilon', \varepsilon_p', z'\right) - U(z') < 0, \\
w(1+g_z)(1+g_p), \text{else}
\end{cases}
$$

The worker earns a real wage of $\frac{w}{p \cdot p_\varepsilon}$, and pays a search cost $C(s, z)$. With probability δ they separate into unemployment. With complementary probability, they remain employed. If they receive an outside offer - which happens with probability $s + \lambda_e$, one of the three cases outlined above will happen. Specifically, they will either (i) receive an offer from a firm which does not constitute a credible threat ($x < q\left(w', y, p', p_\varepsilon', \varepsilon_p', z'\right)$), and their nominal wage will adjust according to w' described above, (ii) receive an offer from a firm which induces a

change in the contracted wage to $\phi^{\text{reneg}}\left(y, x, p', p'_\varepsilon, \varepsilon'_p, z'\right)$, but no job-to-job transition will occur ($q\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right) < x \leq y$), or (iii) receive an offer from a firm which induces a job-to-job transition ($x > y$), where they receive a wage $\phi^{\text{poach}}\left(x, y, p', p'_\varepsilon, \varepsilon'_p, z'\right)$. Finally, the worker may not receive an offer, in which case the wage adjusts to w'. The adjustment to the nominal wage contract before contacts are made is such that (i) the firm is indifferent between keeping the worker and letting the worker go if the real wage is too high, (ii) the worker is indifferent between working and not working if the real wage is too low, and (iii) the nominal wage grows according to the contract otherwise.
Optimal search effort for this worker will satisfy:

$$s_e^*\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) = \begin{cases} 0, & \text{if } \tilde{s}_e\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) < 0, \\ \tilde{s}_e\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right), & \text{if } 0 \leq \tilde{s}_e\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) \leq \bar{s}_e, \\ \bar{s}_e, & \text{if } \tilde{s}_e\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) > \bar{s}_e \end{cases} \tag{2}$$

where

$$\begin{aligned} C_s\left(\tilde{s}_e\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right), z\right) = {} & \beta \mathbf{E}_{\varepsilon'_p}\left(1-\delta\right) \int^{q\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right)} W\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right) dF\left(x\right) \\ & + \beta \mathbf{E}_{\varepsilon'_p}\left(1-\delta\right) \int_{q\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right)}^{y} W\left(\phi^{\text{reneg}}\left(y, x, p', p'_\varepsilon, \varepsilon'_p, z'\right), y, p', p'_\varepsilon, \varepsilon'_p, z'\right) dF\left(x\right) \\ & + \beta \mathbf{E}_{\varepsilon'_p}\left(1-\delta\right) \int_{y}^{\bar{y}} W\left(\phi^{\text{poach}}\left(x, y, p', p'_\varepsilon, \varepsilon'_p, z'\right), x, p', p'_\varepsilon, \varepsilon'_p, z'\right) dF\left(x\right) \\ & - \beta \mathbf{E}_{\varepsilon'_p}\left(1-\delta\right) W\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right) \end{aligned}$$

Optimal search effort will be a function of the cutoff firm $q\left(\cdot\right)$; as $q\left(\cdot\right)$ falls, the share of offers which result in a higher wage increases, raising the returns to search effort.

Given the behavior of workers, we can now derive the value of employment to the firm as:

$$\begin{aligned} J\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) = {} & Y\left(z, y\right) - \frac{w}{p \cdot p_\varepsilon} \\ & + \beta(1-\delta)\lambda_e s_e^*\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) \int_{q\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right)}^{y} J\left(\phi_w^{\text{reneg}}\left(y, x, p', p'_\varepsilon, \varepsilon'_p, z'\right), y, p', p'_\varepsilon, \varepsilon'_p, z'\right) dF\left(x\right) \\ & + \beta(1-\delta)\lambda_e s_e^*\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right) \mathbf{E}_{\varepsilon'_p}\left[\int^{q\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right)} dF\left(x\right)\right] J\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right) \\ & + \beta\left(1-\delta\right)\left[\left(1-\lambda_e s_e^*\left(w, y, p, p_\varepsilon, \varepsilon_p, z\right)\right)\right] \mathbf{E}_{\varepsilon'_p} J\left(w', y, p', p'_\varepsilon, \varepsilon'_p, z'\right) \end{aligned}$$

where

$$p' = p\left(1+g_p\right)$$

$$p'_\varepsilon = p_\varepsilon\left(1+\varepsilon'_p\right)$$

$$z' = \left(1+g_z\right)z$$

$$w' = \begin{cases} \hat{w} : J\left(\hat{w}, y, p', p'_\varepsilon, \varepsilon'_p, z'\right) = 0, \text{if } J\left(w\left(1+g_z\right)\left(1+g_p\right), y, p', p'_\varepsilon, \varepsilon'_p, z'\right) < 0 \\ \hat{w} : W\left(\hat{w}, y, p', p'_\varepsilon, \varepsilon'_p, z'\right) - U\left(z'\right) = 0, \text{if } W\left(w\left(1+g_z\right)\left(1+g_p\right), y, p', p'_\varepsilon, \varepsilon'_p, z'\right) - U\left(z'\right) < 0, \\ w\left(1+g_z\right)\left(1+g_p\right), \text{else} \end{cases}$$

The firm produces output $Y(z,y) = zy$, but must pay its worker $\frac{w}{p \cdot p_\varepsilon}$. It then takes as given the worker's search behavior, $s_e^*(w, y, p, p_\varepsilon, \varepsilon_p, z)$ which determines the probability that the relationship remains intact, and the probability that wages will be renegotiated in the case that it does. In the event of a separation, we assume that the firm earns a value of zero.

2.2 Balanced Growth

Proposition 1 *If (i) $Y(z,y)$, $B(z)$, $C(z,s)$ are homogeneous of degree 1 in z and (ii) wage contracts are indexed to trend inflation g_p and TFP g_z, then*

$$U\Big((1+g_z)\cdot z\Big) = (1+g_z)\cdot U(z),$$

$$W\Big((1+g_z)(1+g_p)\cdot w, y, (1+g_p)\cdot p, p_\varepsilon, \varepsilon_p, (1+g_z)\cdot z\Big) = (1+g_z)\cdot W(w, y, p, p_\varepsilon, \varepsilon_p, z), \quad \textit{and}$$

$$J\Big((1+g_z)(1+g_p)\cdot w, y, (1+g_p)\cdot p, p_\varepsilon, \varepsilon_p, (1+g_z)\cdot z\Big) = (1+g_z)\cdot J(w, y, p, p_\varepsilon, \varepsilon_p, z)$$

Proposition 1 implies that the model scales with TFP growth and trend inflation, so that we can solve the model for a single pair z, p, reducing the state space to nominal wages, firm productivity, the stochastic price level component p_ε, and the shock to the price level ε_p. Critically, the rate of trend inflation does not matter for allocations as long as it is deterministic and wages are indexed appropriately.

2.2.1 Planner's Problem

To understand the efficiency properties of our equilibrium, we consider the allocation that would be chosen by a social planner who can directly control search effort to maximize the joint value of each match:[6]

[6] Another benchmark to consider would instead be the bilaterally efficient degree of search, which is the amount when search is chosen to maximize the joint value of the match from the firm and worker's perspective. In this case, search effort would always be zero, so relative to this benchmark our environment would have excessive search.

$$
\begin{aligned}
M(y,z) = \max_{s^{sp}\in[0,\bar{s}^{sp}]} & Y(y,z) - C(s^{sp},z) + \beta\delta U(z') \\
& + \mathbf{E}_{\varepsilon'_p}\beta(1-\delta)(s^{sp}+\lambda_e)\int_{\underline{y}}^{y} M(y,z')\,dF(x) \\
& + \mathbf{E}_{\varepsilon'_p}\beta(1-\delta)(s^{sp}+\lambda_e)\int_{y}^{\bar{y}} M(x,z')\,dF(x) \\
& + \mathbf{E}_{\varepsilon'_p}\beta(1-\delta)(1-s^{sp}-\lambda_e)M(y,z')
\end{aligned} \tag{3}
$$

and the value of unemployment under the social planner's allocation is:

$$
U(z) = \max_{s_u\in[0,\bar{s}_u]} \left\{ b - C(s_u,z) + \beta(\lambda_u + s_u)\int_{\underline{y}}^{\bar{y}} M(x,z')dF(x) + \beta(1-\lambda_u-s_u)U(z') \right\} \tag{4}
$$

The socially optimal search effort for employed workers is determined by the first-order condition:

$$
\begin{aligned}
C_s(s^{sp}(y,z),z) = & \mathbf{E}_{\varepsilon'_p}\beta(1-\delta)\int_{\underline{y}}^{y} M(y,z')\,dF(x) \\
& + \mathbf{E}_{\varepsilon'_p}\beta(1-\delta)\int_{y}^{\bar{y}} M(x,z')\,dF(x) \\
& - \mathbf{E}_{\varepsilon'_p}\beta(1-\delta)M(y,z')
\end{aligned} \tag{5}
$$

The planner's search choice differs from the worker's individual choice because it accounts for the full impact of search on match value, including both effects on the current firm and future possible values of match output. The first consideration would make the planner lean towards lowering search effort, since the planner cares about the lost value to the firm should the worker's search be successful. The second consideration may depend on the position of the worker in the job ladder. At the bottom of the ladder, private search may be too low, since their are considerable output gains to matching with higher ranked firms in the future which the worker does not internalize. At the top of the ladder, there is only a rent extraction motive to search, so it will be too high relative to a planner's desired amount.

This benchmark allows us to decompose the welfare costs of inflation into: (i) genuine efficiency losses from excessive or insufficient search, and (ii) redistributive effects between workers and firms that do not affect total surplus.

3 Calibration

3.1 Parameterization and Targeted Moments

We calibrate the model to match moments from the pre-COVID US economy spanning 2000-2019, together with moments from the Survey of Consumer Expectations (SCE). One model period is one month. We set the discount rate, β, so that it is consistent with an annual interest rate of 5 percent. We set the deterministic TFP growth rate, g_z, so that it is consistent with an annual growth rate of 0.5 percent, reflecting the average annual growth rate of TFP from 2005 through 2019 [7]. We set trend inflation, g_p, so that it is consistent with an annual inflation rate of 2 percent. Last, we set the job offer arrival rate of unemployed workers, λ_u, to be 0.34 so that the model delivers a job-finding rate that is consistent with the average UE transition rate over the same time period.

As laid-out in the model, the inflation shock process is AR(1) with parameters ρ and σ_ν. We estimate these parameters using the entire history of the CPI in the US from 1947 onward. First, we obtain the cyclical component of annual inflation, converted to a monthly frequency, using the HP filter with a smoothing parameter of 14400. Second, we estimate ρ and σ_ν from the autoregressive model of the cyclical component of inflation. This yields an estimate of ρ of around 0.936 and an estimate of σ_ν of around 0.00036. In practice, we discretize the continuous inflation shock process ε_p characterized by ρ and σ_ν^2. We use the Rouwenhorst method to approximate the process using 7 grid points [8].

For our quantitative exercise, we extend the model in three key dimensions. First, we allow the job destruction rate faced by workers to depend on firm productivity. That is, we assume that the relationship is linear such that $\delta(y) = \delta_0 + \delta_1 y$. This embeds a "slippery" job ladder into our model, which helps in matching wage growth moments well. We target δ_0 to match the average separation rate of 0.025 from the SCE, yielding an estimate of δ_0 around 0.0175. We target δ_1 to match the separation-wage elasticity of -0.0392 documented by Jung and Kuhn (2019) using the SIPP, yielding an estimate of -0.024. Second, we allow there to be exogenous job mobility shocks ("godfather shocks"), λ_g, under which a worker draws a job offer and either can accept the offer or receive the value of unemployment. Following the approach of Faberman et al. (2022), we discipline λ_g using the offer acceptance ratio between employed and non-employed workers, as measured in the SCE. The key intuition is that, all else constant, λ_g will tend to increase the share of offers accepted by employed workers, but not non-employed workers. Finally, we introduce moving costs η_e for EE transitions and η_u for UE transitions as

[7] The TFP series is drawn from Fernald (2014). The series adjusts for variable utilization of capital and labor inputs, providing a more measure measure of TFP growth.

[8] To capture larger inflationary shocks well, we first use the Rouwenhorst method to approximate the process using 140 grid points. We then collapse the associated transition matrix and grid down to 7 grid points. This allows the tails of the discrete distribution to be more extreme, but with appropriate transition probabilities.

described in Appendix XX .[9] For the moving costs, we target the average 12-month real wage growth of job movers [10] and job movers who experience a job separation. The 12-month real wage growth of job movers is measured from administrative payroll data as 0.06 (Grigsby et al., 2021). The 12-month real wage growth of job movers who experience a job separation is measured from the SIPP as -0.022 (Fujita and Moscarini, 2017). Intuitively, if moving costs are higher, workers are more selective in the job offers they take: conditional on moving, wage growth will be higher the higher are moving costs.

We assume that the search cost function takes the form $C(s) = C_0 s^{\kappa}$, $\kappa > 1$. We calibrate κ to match the search effort-wage elasticity of -0.07 documented by Faberman et al. (2022) using the Survey of Consumer Expectations (SCE), where search effort is captured by the decision to submit a job application. Our estimate for κ is 1.92, implying that search costs are approximately quadratic. We calibrate C_0 to match the relative offer yield ratio between employed and unemployed workers of 0.237 documented by Faberman et al. (2022) using the SCE, finding that C_0 is 15.76. Intuitively, the relative offer yield ratio between employed and unemployed workers is closely related with C_0, as unemployed workers will never put forth search effort in our model and search effort is decreasing in C_0 for employed workers. We use the average monthly job-to-job transition rate, found to be 0.0241 from 2000 through 2019 by Fujita et al. (2024), to discipline the job offer arrival rate, λ_e, faced by employed workers. Our estimate for λ_e is 0.055, which is substantial in magnitude, reflecting the fact that employed workers reject many outside offers, as most workers are employed at the top of the firm productivity distribution.

We assume that the exogenous vacancy distribution across firm types is a truncated Beta so that $y \sim Beta(\tilde{\alpha}, \tilde{\beta})$, with support $[b, 1]$. In order to discipline parameters governing the vacancy distribution, we target dispersion in firm fixed effects[11] and the average 12-month real wage growth of job stayers respectively. The empirical moments we target are measured from administrative payroll data as 0.019 for wage growth of job stayers and 0.032 for dispersion of firm fixed effects (Grigsby et al., 2021; Lachowska et al., 2023). Our estimate for $\tilde{\alpha}$ is 3.31 and our estimate for $\tilde{\beta}$ is 2.39.

Finally, to calibrate the flow value of unemployment b, we target the replacement rate of average flow output, $\frac{b}{\mathbb{E}[y]}$, using a target of 0.55 consistent with exercises in Chodorow-Reich and Karabarbounis (2016). Given that the wide range of structural estimates of the replacement rate found in the literature, we place comparatively less weight on this target, allowing for wage growth moments to inform b through its influence on the vacancy distribution. This yields an

[9]The value functions we use for the quantitative model are all spelled out in the same Appendix.

[10]The 12-month average real wage growth of job movers encompasses both workers who experience a non-employment spell before being hired by a new firm, along with workers who experience a job-to-job transition

[11]In order to measure dispersion in firm fixed effects in simulated data, we recover fixed effects from a regression of log wages on a firm dummy, and then compute the variance of the estimated firm dummy coefficients.

estimate of b near 0.53.

Table 1 displays our parameter estimates, along with the data source underlying each targeted moment. Table 2 displays model moments together with targeted moments, providing a sense of the fit of the model.

Table 1: Calibrated Parameter Values

Parameter	Description	Value	Source/Target
A. Externally calibrated parameters and normalizations			
β	Discount rate	0.996	Annual interest rate of 5%
g	Det. TFP growth rate	0.0004	Avg. TFP growth: 0.5%
λ_u	Job arrival rate, unemployed	0.342	SCE, 2001-2019
g_p	Trend inflation	0.0017	CPI/PCE inflation (2000-2019)
ρ	Persistence of inflation shock	0.936	Cyclical component of CPI (1947-2024)
σ_ν	SD of inflation shock	0.00036	Cyclical component of CPI (1947-2024)
B. Internally calibrated parameters			
κ	Elasticity of search cost	1.92	Search-wage elasticity
c_0	Search cost parameter	15.76	Relative offer yield ratio
λ_e	Offer arrival rate of emp.	0.055	EE transition rate
b	Flow value of unemployment	0.829	Replacement rate
δ_0	Intercept of job destruction	0.0175	EU rate
δ_1	Slope coefficient of job destruction	-0.024	Separation-wage elasticity
λ_g	Exogenous job mobility shock	0.009	Acceptance ratio
η_u	Moving cost share of non-employed	0.83	Avg. wage growth of EUE movers
η_e	Moving cost share of employed	0.15	Avg. wage growth of movers
$\tilde{\alpha}, \tilde{\beta}$	Parameters governing vacancy dist.	3.31,2.39	Var. of firm FEs, Avg. wage growth of stayers

Our model accurately captures the acceptance ratio, where workers accept about 31.4% of job offers in the model, closely matching the empirical observation of 32.8%. The employment-to-unemployment (EU) separation rate in our model is 2.3%, slightly below but still close to the empirical target of 2.5%. Importantly, our model successfully replicates the negative separation-wage elasticity documented in Jung and Kuhn (2019), indicating that higher-wage workers have lower separation rates.

The model performs exceptionally well in matching wage dynamics across different labor market transitions. This is a key moment for our model, since our theory ties together search effort, inflation, and wage growth. For job stayers, we precisely match the annual real wage growth of 1.9%. For job movers, our model exactly replicates the empirical 6% annual real wage growth, capturing the substantial wage premium associated with voluntary job transitions. We also closely match the wage penalty experienced by workers who move through unemployment (EUE transitions), with our model generating a 1.98% wage decline compared to the empirical target of 2.2%.

Finally, our model generates a realistic degree of wage dispersion across firms, with a standard deviation of firm fixed effects of 0.037 comparable to its empirical counterpart of 0.032, drawn from Lachowska et al. (2023).

Table 2: Targeted Moments in the Data and the Model

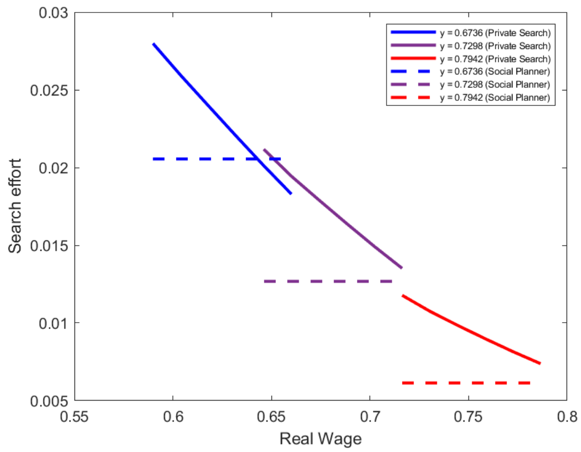
Moment	Source	Data	Model
Search-wage elasticity	Faberman et al. (2022)	-0.07	-0.066
Offer yield	Faberman et al. (2022)	0.237	0.234
Acceptance ratio	Faberman et al. (2022)	0.328	0.314
EE transition rate	Faberman et al. (2022)	0.025	0.028
Replacement rate	Chodorow-Reich et al. (2016)	0.55	0.54
EU separation rate	Faberman et al. (2022)	0.025	0.023
Separation-wage elasticity	Jung and Kuhn (2019)	-0.0392	-0.0393
Real wage growth (job stayers)	Grigsby et al. (2021)	0.019	0.019
Real wage growth (job movers)	Grigsby et al. (2021)	0.06	0.06
Real wage growth (EUE movers)	Fujita and Moscarini (2017)	-0.022	-0.0198
Dispersion in Firm FEs	Lachowska et al. (2023)	0.032	0.037

3.2 Private vs. Social Search Policy Choices

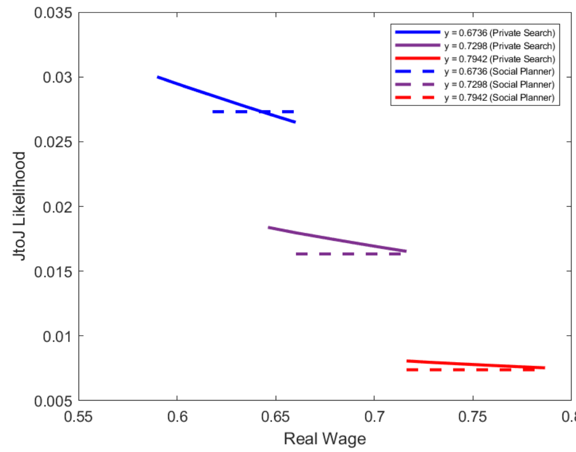
In this section, we analyze how a planner's search policy contrasts with the privately optimal search decisions made by workers in our baseline model. To illustrate this difference, Figure 2 plots the socially optimal search effort and resulting likelihood of job-to-job (J2J) transitions across varying levels of the real wage at a given firm, based on numerical results from our calibrated model. When search effort is privately chosen, workers intensify search efforts in response to reductions in their real wages driven by unanticipated inflation shocks. This creates a negative relationship between real wages and search effort, as workers attempt to mitigate purchasing power losses by obtaining outside offers that can be used to adjust their wage.

The planner, however, makes search decisions that maximize total match surplus rather than individual worker utility, so the search policy is invariant to real wage levels within a given firm type. This invariance arises because the planner internalizes both the costs and benefits of search across all agents. From the planner's perspective, wage levels represent transfers between workers and firms within a match, but do not affect the fundamental trade-off between search costs and the expected productivity gains from worker reallocation. The planner focuses solely on whether additional search effort generates sufficient expected improvements in match quality to justify the social cost of search.

This comparison reveals a key inefficiency in the decentralized equilibrium: workers' search responses to inflation-induced real wage declines generate welfare losses that would be avoided under the social planner's search policy. While workers increase search when the purchasing power of their wage falls, the social optimum calls for search decisions that are independent of these distributional concerns, suggesting that inflation's welfare costs partly stem from the misalignment between private and social search incentives. Conversely, when inflation shocks are negative, workers are searching too little.



(a) Search effort policy function



(b) Implied JtoJ likelihood

Figure 2: Search, Job-to-Job Transitions and the Inflationary Environment

Note: Panel (a) plots the planner's optimal search effort across different real wage levels for workers employed at firms with different productivity levels, showing that socially optimal search is invariant to wage levels within each firm type. Panel (b) plots the corresponding job-to-job transition probabilities under the social planner's policy, which similarly remain constant across real wage levels for each firm productivity type.

4 The Search Costs of Inflation

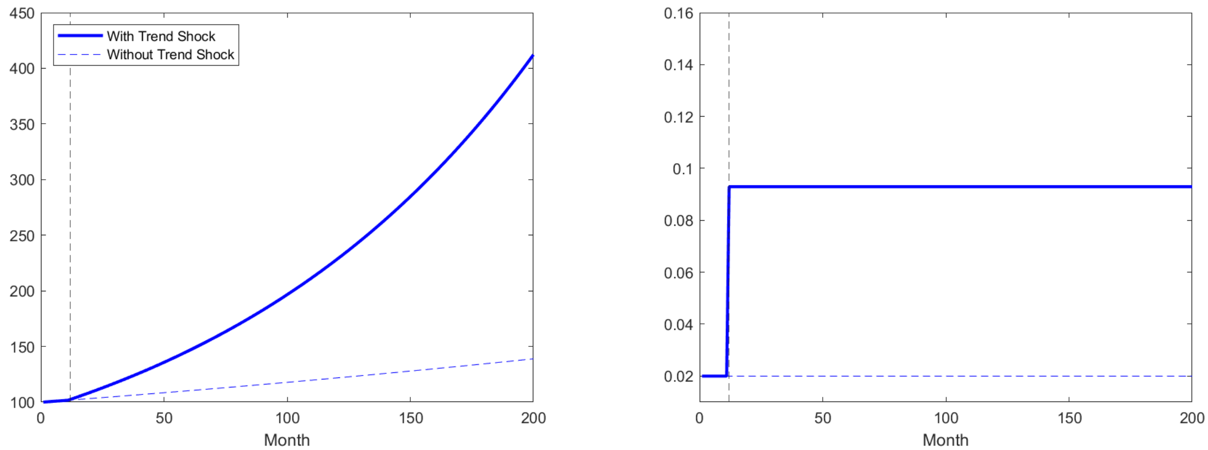
4.1 Trend Inflation Shock

We first consider a one time, unanticipated, but permanent movement in the trend rate of inflation from g_p^{low} to g_p^{high} at some date τ. Due to the balanced growth result, both steady state allocations and search behavior are the same in the two trend inflation regimes; permanent differences in trend inflation are not costly, but the transition path from one regime to the other will entail costs. Specifically, since the cost of living adjustment (COLA) in wage contracts is set according to the prevailing rate of trend inflation, any movements in trend inflation that occur after a contract is signed result in unanticipated real wage declines. These declines are costly, and induce additional costly search behavior.

To implement this shock, we set g_p^{low} to be at an annualized rate of 2 percent as in our calibration, and g_p^{high} to be at an annualized rate in excess of 9 percent, around the maximum of year-over-year inflation during the COVID-19 period in the United States [12]. Figures 3a and

[12]To consider the shock, we draw 4000 ε_p paths. For each ε_p path, we simulate a corresponding economy populated by 10000 workers in the run-up to the shock and afterwards. In order to compute time-series of interest, we first compute time-series of interest at the economy level. We then average over economy-level time-series averages to produce the key time-series discussed in this section.

3b plot the price level (normalized at time $\tau - 1$) and inflation rate, respectively, and highlight how the increase in trend inflation is fully absorbing, resulting in a dramatic increase in the CPI relative to the previous trend.



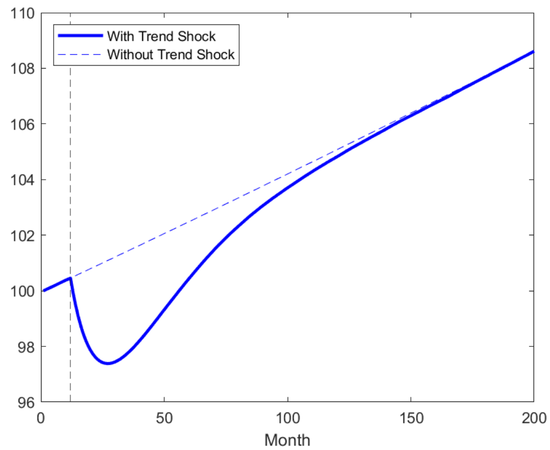
(a) Consumer Price Index

(b) Inflation rate

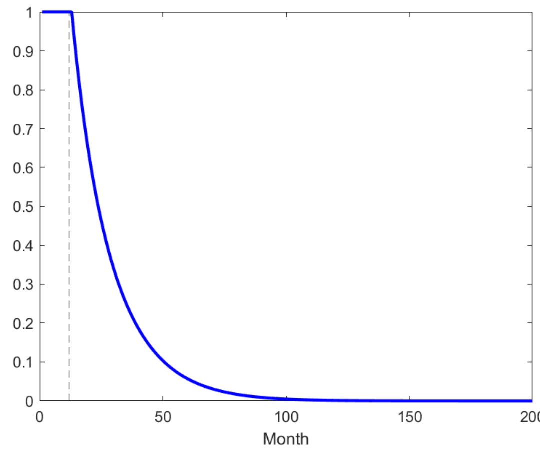
Figure 3: Graphical Illustration of Trend Inflation Shock

Note: Figure 3a plots the realized price index at time t, normalized relative to the first period of the plot, in both the environment absent the trend shock (dotted line) and the counterfactual which introduces the trend shock (solid line). Figure 3b plots the corresponding implied inflation rate in the two scenarios, highlighting the magnitude of the shock.

The increase in trend inflation results in a large decrease in real wages on impact, as contracts are indexed at a lower level of inflation g_p^{low}. As depicted in Figure 4a, in the two years following the shock, real wages decline by nearly five percentage points relative to the world without the trend inflation shock. As wage contracts are re-negotiated, either as new matches are formed, or in existing matches through outside offers or upward wage adjustments, they are negotiated with COLAs indexed to g_p^{high}, preventing wage erosion in the future. Figure 4b illustrates that this process is not immediate and unfolds over six years before it concludes, facilitating a swift increase in the average real wage towards the trend path of labor productivity.



(a) Normalized Average Real Wages



(b) Share of employed workers with g_p^{low} COLA

Figure 4: Real Wages and COLAs in Wage Contracts

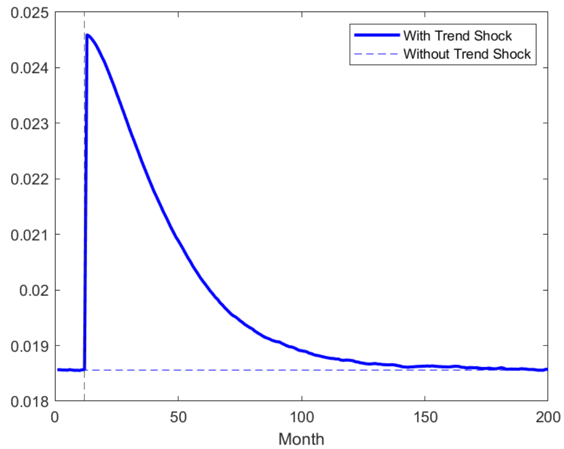
Note: Figure 4a plots the index of average real wages time t, normalized relative to $\tau - 1$, along with the same object in the absence of the trend shock. Figure 4b plots the corresponding share of workers with wage contracts entailing a COLA negotiated at g_p^{low}.

Figures 5a and 5b, illustrate how search effort brings about these wage changes. On impact, average search effort in the economy nearly doubles. This increase occurs through two distinct channels. First, the value of a job with an old COLA contract diminishes substantially relative to the value of a job with a new contract, immediately increasing the incentive to put forth more search effort. Second, over time, real wages of workers with old COLA contracts are eroded, which further increases the returns to putting forth more search effort. As a result, search effort for workers with an outdated COLA contract continuously increases in each period as real wages erode further. In contrast, the average level of search effort of workers with new COLA contracts is markedly lower, even though a disproportionate share of these contracts are newly formed out of unemployment [13].

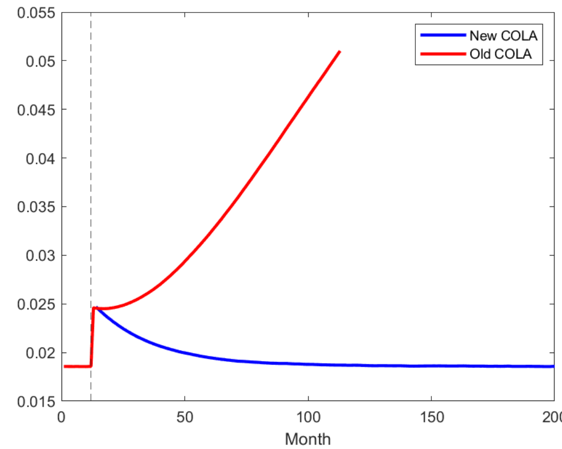
As our earlier comparative statics analysis showed, the increase in search effort implies an increase in both the job-to-job transition rate (Figure 6a) and the renegotiation rate (Figure 6b). Therefore, inflationary shocks eventually induce an increase in wages that can be purely redistributive or entail productivity growth, as depicted in Figure 9a. As more productive jobs are more stable jobs, the unemployment rate falls, an outcome that mirrors the findings of Blanco and Drenik (2023).[14]

[13] As depicted in Section C of the Appendix, initially, workers in new COLA contracts typically have lower real wages and this is why their average search effort is higher.

[14] In Blanco and Drenik (2023), this occurs through inflationary pressures moving the distribution of real wages away from an endogenous separation threshold. In our model, this occurs through workers reallocating up the job ladder, which is associated with a reduction in the separation rate. Through both channels, the increase



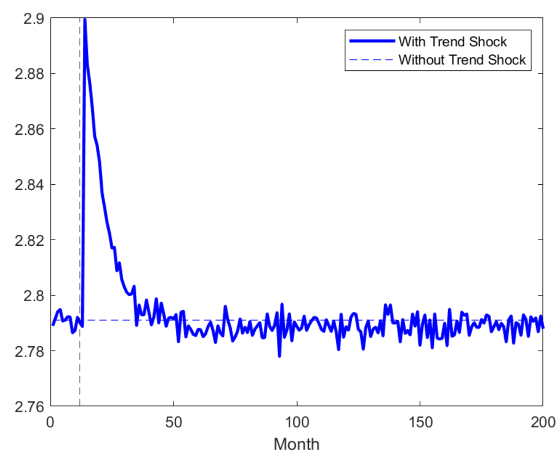
(a) Average search effort



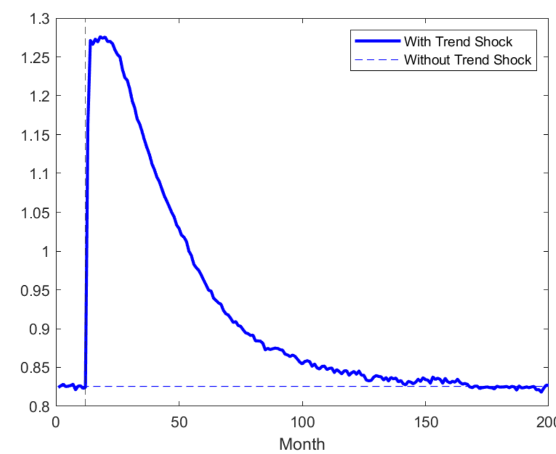
(b) Average search effort by contract type

Figure 5: Real Wages and Wage Contract Types

Note: Figure 5a plots average search effort in the economy, along with a dashed line indicating the average level of search effort in steady-state. Figure 5b plots average search effort exerted by workers in contracts with g_p^{low} COLAs and average search effort exerted by workers in contracts with g_p^{high} COLAs.



(a) Job-to-Job transition rate

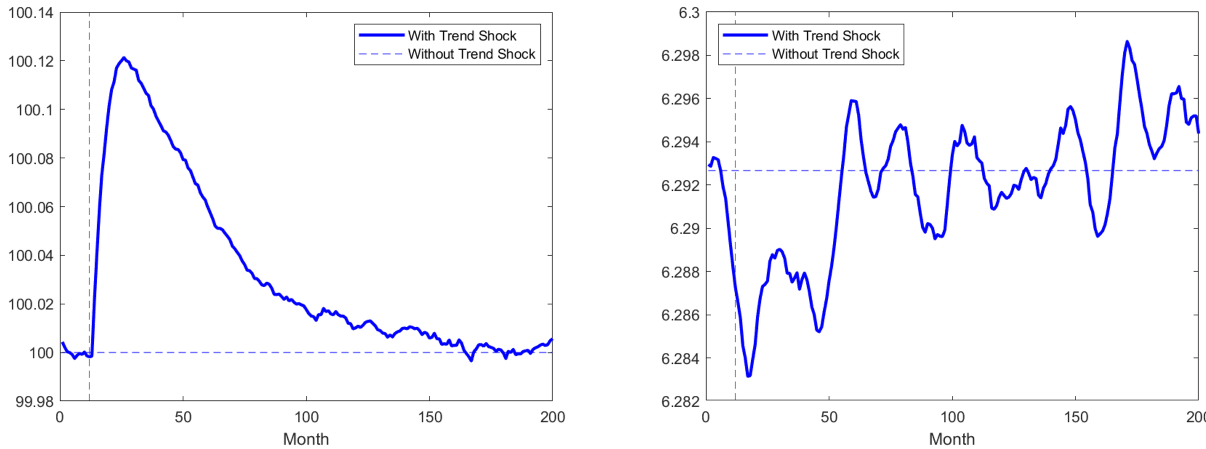


(b) Renegotiation rate for job stayers

Figure 6: Mechanisms of Adjustment

Note: Figure 8a plots the Job-to-Job transition rate, along with a dashed line indicating the steady-state Job-to-Job transition rate. Figure 8b plots the rate at which employed workers who are job stayers renegotiate their contracts using outside offers, along with a dashed line indicating the steady-state renegotiation rate

in trend inflation results in a temporary increase in output per person.



(a) Detrended, normalized output per worker (b) Unemployment rate

Figure 7: Reallocative Consequences of the Shock to Trend Inflation

Note: Figure 9a plots the level of detrended output per-worker, normalized relative to $\tau - 1$, along with a dashed line indicating the steady-state . Figure 8b plots the rate at which employed workers renegotiate their contracts using outside offers, along with a dashed line indicating the steady-state renegotiation rate

4.2 Welfare Costs: Trend Inflation Shocks

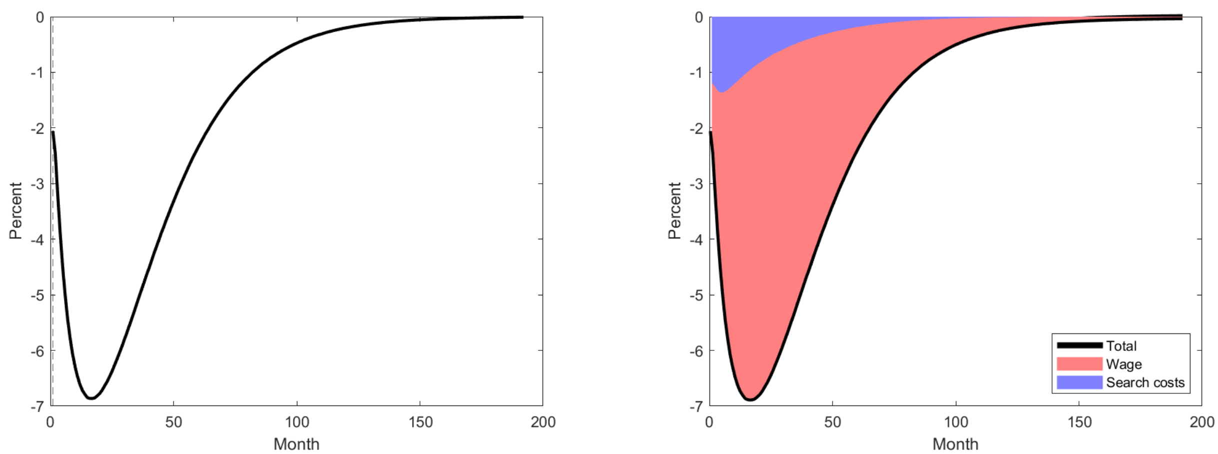
While the economy converges to its baseline allocations following the the trend inflation shock, the adjustment process has important implications on the welfare of agents in the economy along the transition. First, the shock lowers real wages on impact, a direct negative cost to workers. Second, and as a result, workers pay a disutility cost when they respond by increasing their search effort. This latter cost is a deadweight loss to the economy, whereas the real wage loss is a net benefit to firms. On the other hand, due to their efforts, workers are temporarily reallocated up the job ladder, resulting in a decrease in the unemployment rate and an increase in labor productivity. Therefore, in general, given the mechanisms of our model, the effect of the trend inflation shock on the welfare of agents in the economy is ambiguous.

We quantify the effect of the trend inflation shock on the welfare of agents in the economy, beginning with workers. To do so, we simulate a counterfactual economy that does not face the trend inflation shock, and compare the flow value of income less search costs for each worker in the two economies over time.[15]

Figure 8a displays the average flow utility loss in each period (income less search costs in the economy with the shock relative to the economy without the shock) where the first month plotted

[15] The overall cost would be a weighted sum of the per period costs we plot. However, since agents are infinitely lived, expressing the costs this way would mute the effect, since eventually the economy returns to the same allocation.

is τ, the time when the shock is first transmitted . On impact, average flow losses are large, even as real wages are almost unchanged in the first period: this occurs because search effort immediately increases in response to the shock to trend inflation, as workers expect inflation to remain elevated in the future. These flow losses increase substantially over time as the real wages of workers in old contracts continue to erode; roughly two years after the shock, flow losses reach their maximum at around 7 percent. Thereafter, while the losses begin to reverse, they remain significantly negative 10 years after the shock. This is true despite the fact that only a small share of workers are employed with "old" contracts 5 years later; they are still poorly allocated on the job ladder. Therefore, from the perspective of a worker, a large unexpected shock to trend inflation rate is very costly.



(a) Worker welfare losses

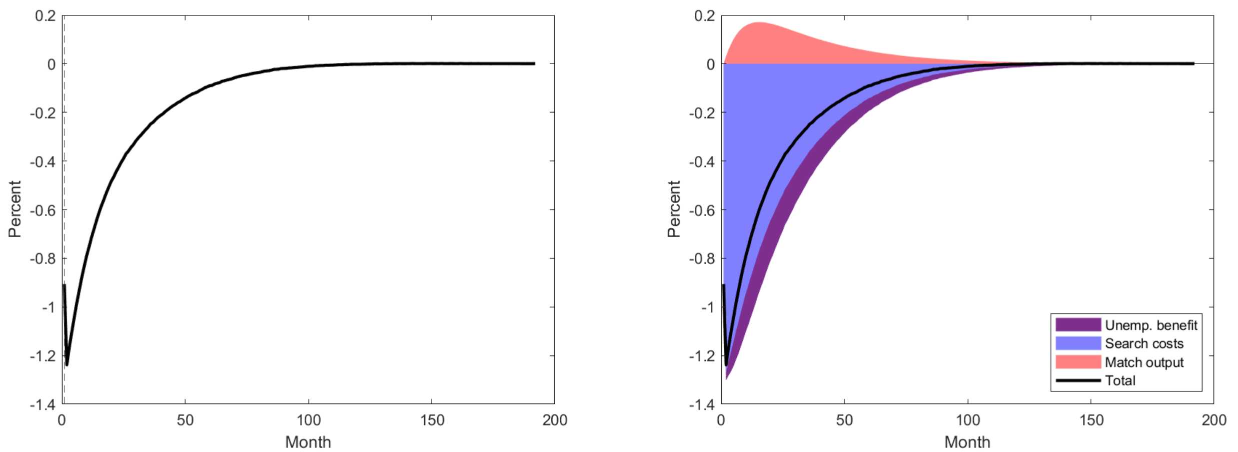
(b) Decomposition of worker welfare losses

Figure 8: Per-period Welfare Costs of a Shock to Trend Inflation: Workers

Note: Figure 8a plots the difference between the average real wage net of search costs in the shocked economy and the baseline economy in percentage terms. Figure 8b decomposes the sources of this percentage difference into the amount explained by differences in real wages, represented by the red shaded area, and the amount explained by differences in search costs, represented by the blue shaded area.

In order to consider the *aggregate* welfare costs induced by the trend inflation shock, we additionally consider average flow output net of search costs (i.e. flow match value). Figure 9a displays the aggregate flow losses experienced by agents in the shocked economy per period, where the first month plotted is the first month of the shock. Per-period average flow losses are naturally lower than the corresponding losses for workers, since firms gain from the lower real wage. However, the aggregate costs remain large, with 1 percent flow loss on impact. As contracts readjust, per-period losses gradually diminish, becoming negligible relative to baseline around 8 years later. The shock to trend inflation rate is quite costly to the economy as a whole, with the search costs of inflation dominating the reallocation channel.

As shown in Figure 9b, which decomposes the sources of the per-period losses faced by agents into a portion explained by (i) differences in real average match output (ii) differences in average search costs and (iii) differences in unemployment benefits (explained by differences in employment rates), most of the per-period welfare losses are experienced by agents in the form of increased search costs as seen in the blue shaded region. The reallocation of workers towards better firms that occurs due to increased search effort only results in a small increase in real output.



(a) Aggregate welfare losses

(b) Decomposition of aggregate welfare losses

Figure 9: Aggregate Per-period Welfare Costs of a Shock to Trend Inflation

Note: Figure 9a plots the difference between the average output per worker net of search costs in the shocked economy and the baseline economy in percentage terms. Figure 9b decomposes the sources of this percentage difference into the amount explained by differences in real output, represented by the red shaded area, the amount explained by differences in search costs, represented by the blue shaded area, and the amount explained by differences in home production, represented by the purple shaded area.

In order to consider the *lifetime* welfare costs of the inflationary shock, we compute the present discounted value (PDV) of worker flow values net of search costs for each worker, $\mathcal{W}_i$, in both the shocked economy and baseline economy, from the first period of the shock onward, which amounts to integrating over the per period losses depicted in Figure 8a. We do the same for the aggregate losses, and the results are listed in Table 3.

Table 3 summarizes the average lifetime welfare losses induced by the shock to trend inflation. The average PDV of workers in the shocked economy is nearly 1% lower than the PDV of workers in the baseline economy, with around 85 percent of this difference being experienced in the form of lower real wages and 15 percent being experienced in the form of real search costs. As lower real wages result in higher employer profits, the average aggregate welfare losses induced by the inflationary shock are around 0.122% of the PDV of real output in the baseline economy, with search costs dominating the small increase in output in the shocked economy induced by increased search effort.

Table 3: Lifetime Welfare Consequences of a Shock to Trend Inflation

	Change
Worker welfare changes (percent of baseline)	-1.35%
Wage channel	-1.18%
Search cost channel	-0.15%
Overall welfare changes (percent of baseline)	-0.106%
Output channel	0.03%
Search cost channel	-0.13%
Unemp. flow value channel	-0.004%

Note: Row 1 depicts the value of expression 6. Subrows decompose this difference in PDVs into a component explained by the difference in PDV of real wages and a component explained by the difference in PDV of search costs. Row 2 depicts the value of expression 8. Subrows decompose this difference in PDVs into a component explained by the difference in PDV of real output and a component explained by the difference in PDV of search costs.

4.3 Heterogeneous Effects Across the Wage Distribution

4.3.1 Wage Compression and Differential Adjustment

A striking feature of the trend inflation shock is its profound impact on wage inequality. As illustrated in Figure 10, the variance of log wages falls from 0.0555 to approximately 0.051 within two years of the shock, representing a nearly 8 percent reduction in wage dispersion. This compression persists even as the economy adjusts to the new inflation regime.[16]

[16]Figure 10 displays the evolution of wage dispersion following the trend inflation shock, demonstrating the immediate and persistent compression in the wage distribution.

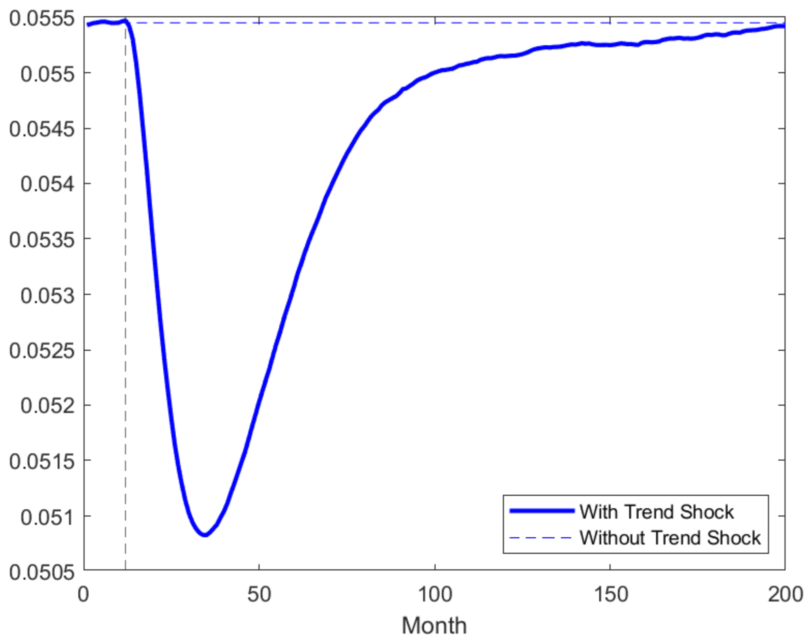


Figure 10: Wage Dispersion Following Trend Inflation Shock

Note: This figure displays the variance of log wages over time. The vertical dashed line indicates the period of the trend inflation shock. The substantial and persistent decline demonstrates wage compression.

To understand the mechanisms driving this compression, Figure 11 decomposes the evolution of real wages by initial wage quartile. Workers in the highest wage quartile experience the most severe and persistent erosion of real wages, falling to 95.5 percent of their pre-shock level and remaining depressed for over a decade. In stark contrast, workers in the lowest quartile see their real wages decline by only 2 percent at the trough, with a recovery beginning within two years.[17]

[17] Figure 11 tracks average real wages by wage quartile, indexed to 100 in the period before the shock. The differential impact across quartiles drives the observed wage compression.

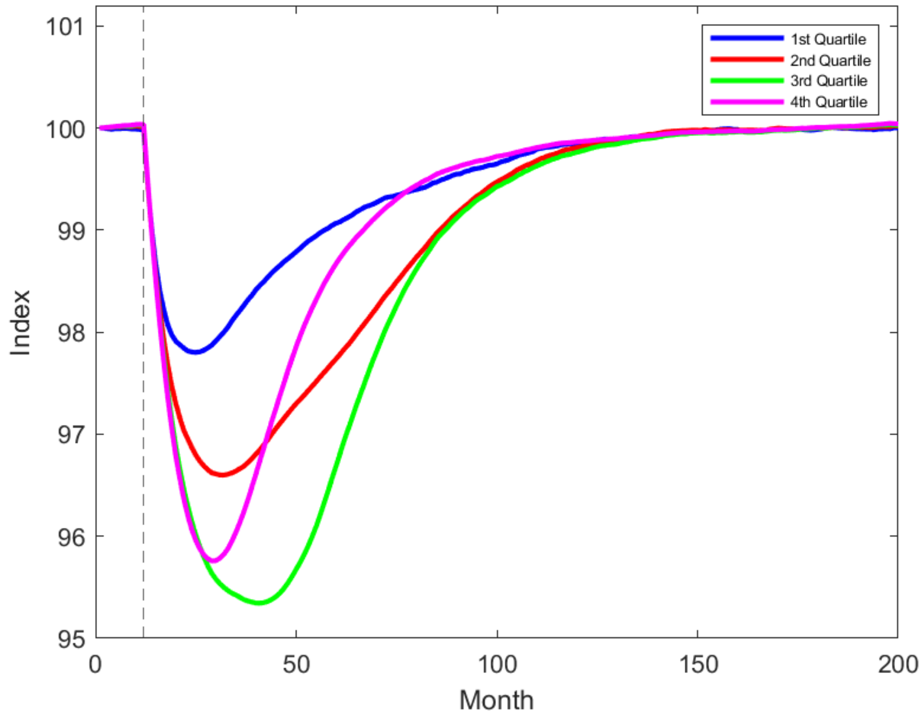


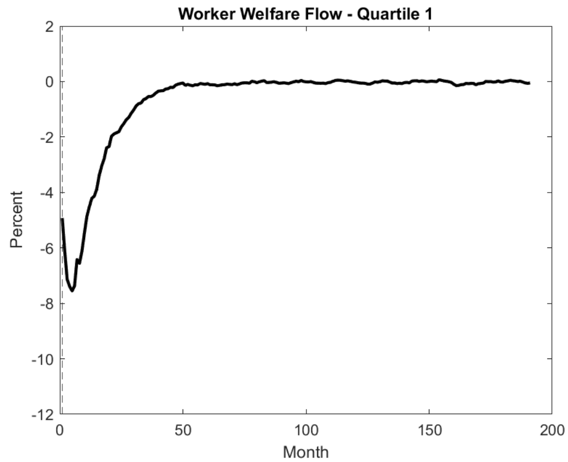
Figure 11: Real Wages by Wage Quartile

Note: This figure plots average real wages by initial wage quartile, indexed to 100 at $\tau - 1$. Workers are sorted into quartiles based on their real wage in the period before the shock. The vertical dashed line indicates the timing of the trend inflation shock.

This differential adjustment reflects fundamental differences in labor market dynamics across the wage distribution. Workers at the bottom of the wage distribution face higher job destruction rates and are more likely to receive outside offers that trigger wage renegotiations. Following the balanced growth properties of our model, workers employed at less productive firms—who tend to earn lower wages—have both higher separation rates $\delta(y)$ and a larger set of firms from which they would accept offers. Consequently, their wage contracts adjust more rapidly to the new inflation regime through both job transitions and renegotiations with incumbent employers.

4.3.2 Heterogeneous Welfare Impacts

While all workers in our model are ex-ante homogeneous, their position in the firm productivity distribution at the time of the shock generates substantial heterogeneity in welfare impacts. Figure 12 compares the per-period welfare losses for workers in the lowest and highest wage quartiles.



(a) First Quartile (Lowest Wages)

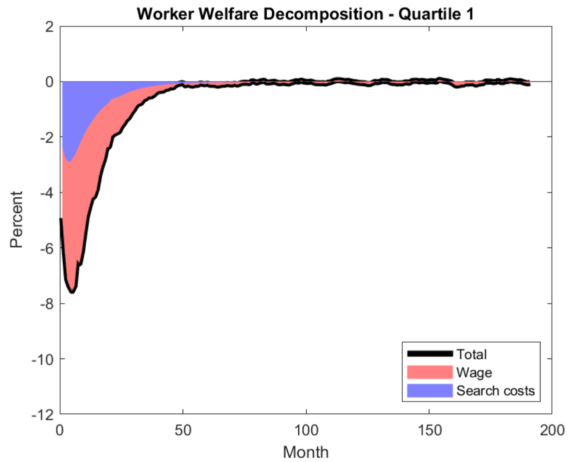
(b) Fourth Quartile (Highest Wages)

Figure 12: Worker Welfare Losses by Wage Quartile

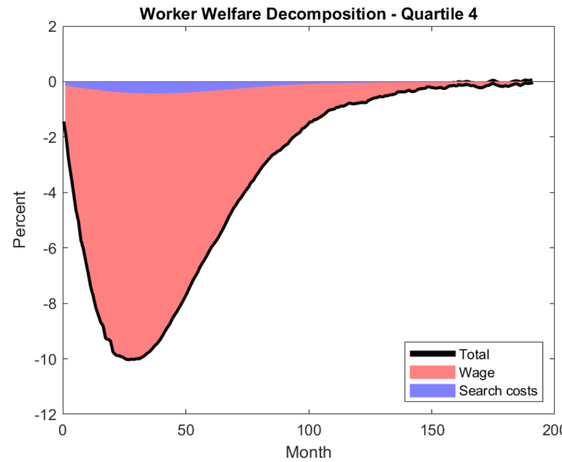
Note: These figures plot per-period welfare losses (real wages net of search costs) relative to the counterfactual economy without the trend shock, expressed as a percentage of baseline flow utility.

Workers in the lowest wage quartile experience severe but transitory welfare losses, reaching nearly 7 percent at the peak before recovering within five years. In contrast, workers in the highest quartile face smaller peak losses of approximately 1 percent, but these losses persist for over a decade. This pattern reflects a fundamental trade-off: low-wage workers can escape the erosion of their purchasing power more quickly through job mobility, but they must incur substantial search costs to do so. The decomposition of these welfare losses, shown in Figure 13, reveals strikingly different adjustment mechanisms across the wage distribution. For workers in the lowest quartile, search costs (the blue shaded area) dominate the welfare losses in the immediate aftermath of the shock, reflecting their dramatic increase in search intensity. The wage component of their losses is relatively short-lived, consistent with their rapid contract adjustment. For workers in the highest quartile, wage losses constitute the predominant and persistent component of welfare losses, as their limited job mobility leaves them trapped in contracts with inadequate cost-of-living adjustments.[18]

[18]Additional figures showing search effort, job-to-job transitions, and wage renegotiation rates by quartile are provided in Appendix Section E.3.



(a) First Quartile (Lowest Wages)



(b) Fourth Quartile (Highest Wages)

Figure 13: Decomposition of Worker Welfare Losses

Note: These figures decompose per-period welfare losses into components attributable to real wage declines (red shaded area) and increased search costs (blue shaded area). The black line shows total welfare losses.

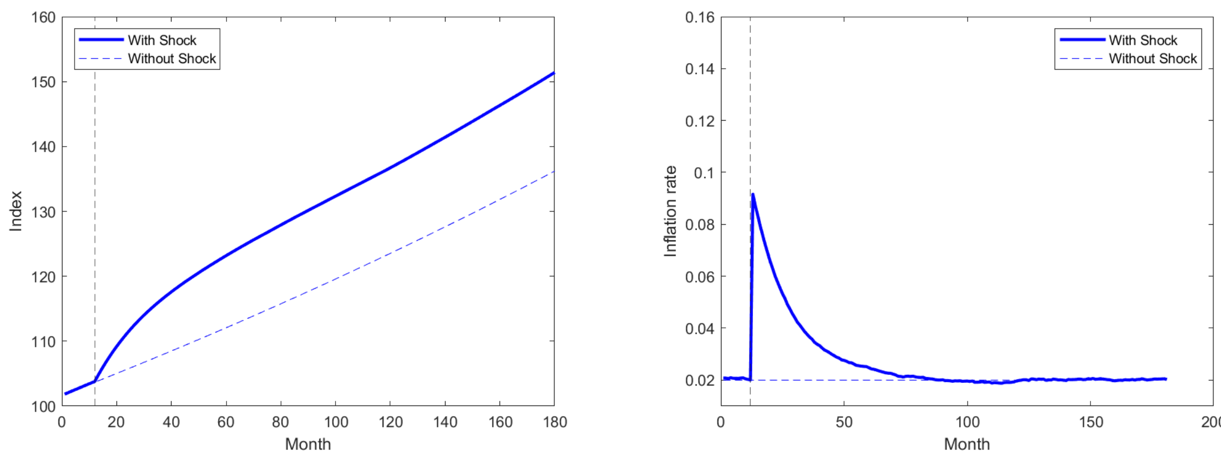
These heterogeneous effects highlight an important distributional consequence of unexpected inflation: while wage compression might appear to reduce inequality, it masks substantial differences in adjustment costs and the persistence of welfare losses. Workers at the bottom of the wage distribution bear the burden of adjustment through costly search effort, while those at the top experience prolonged erosion of their purchasing power. The net effect is a flattening of the wage distribution that comes at considerable cost to workers throughout the distribution, albeit through different channels.

4.4 Cyclical Inflation Shocks

While the trend inflation shock provides insights into the welfare costs of persistent monetary regime changes, the analysis of cyclical inflation shocks offers a complementary perspective on how agents respond to temporary but persistent inflationary episodes. The key distinction lies in the expectations surrounding shock persistence: whereas the trend shock represents a permanent shift in the inflation environment, the ε_p shock is non-absorbing, with agents anticipating its gradual reversion to mean according to the underlying stochastic process. This temporal dimension generates two important behavioral differences. First, because workers expect the inflationary pressure to diminish over time, they moderate their search effort response relative to what would be optimal under a permanent shock, recognizing that the real wage erosion they face will naturally attenuate. Second, unlike the trend shock environment where wage contracts can eventually be renegotiated around the new inflation regime, the temporary nature of ε_p shocks requires agents to continually adapt their contracting behavior as the shock evolves, with workers obtaining higher negotiated wages precisely when ε_p is elevated.

To examine these mechanisms, we quantify the welfare costs of a large, unexpected and persistent inflationary shock, where ε_p suddenly moves from its mean to an annualized inflation rate of ε^{shock} (9 percent) at period τ.[19] We then allow the future expected path of ε_p to evolve in accordance with the underlying inflation process, which agents anticipate.[20] We measure the welfare costs using the same methodology as in the previous section.

Under the cyclical inflation shock, the annualized inflation rate declines quite rapidly, falling from around 15 percent to around 5 percent three years later, finally returning to its mean level after about five years. Figure 14b provides a graphical illustration of the expected path of inflation $g_p + \varepsilon_p$, given the inflationary shock.



(a) Consumer Price Index

(b) Inflation rate

Figure 14: Graphical Illustration of Inflationary Shock

Note: Figure 14a plots the realized price index at time t, normalized relative to the first period of the plot, along with the trend path of prices. Figure 14b plots the inflation rate at different periods, highlighting the magnitude of the shock.

As laid-out in Section 4.2, we compute the average lifetime welfare losses induced by the large shock to ε_p [21] . Table 4 summarizes the average lifetime welfare losses induced by the

[19] While agents believe that a shock of this magnitude is possible, it is exceedingly unlikely. In this sense, it is appropriate to describe the shock as being unexpected.

[20] In practice, we simulate a large set of economies under different inflation paths. To do this, we draw a set of inflation paths where ε_p is equal to its mean in $\tau - 1$ and then equal to ε^{shock} at τ, drawing future inflation paths for T periods given the transition matrix governing ε_p. We simulate all objects of interest implied by our model under each inflation path, taking the average over inflation paths to consider the effect of the large, persistent inflationary shock.

[21] We consider the dynamics of per-period welfare losses under the inflationary shock process in Section C.2

shock. The average PDV of workers in the shocked economy is 0.7% lower than the PDV of workers in the baseline economy, with around 89 percent of this difference being experienced in the form of lower real wages and 11 percent being experienced in the form of real search costs. As lower real wages result in higher employer profits, the average aggregate welfare losses induced by the shock to ε_p are around 0.068 percent of the PDV of real output in the baseline economy, with search costs dominating the small increase in output in the shocked economy induced by increased search effort.

Table 4: Lifetime welfare consequences of ε_p shock

	Change
Worker welfare changes (percent of baseline)	-0.83 %
Wage channel	-0.74 %
Search and moving cost channel	-0.08 %
Unemp. flow value channel	0.006 %
Overall welfare changes (percent of baseline)	-0.051 %
Output channel	0.012 %
Search cost channel	-0.069 %
Unemp. flow value channel	0.006 %

Note: Row 1 depicts the value of expression 6. Subrows decompose this difference in PDVs into a component explained by the difference in PDV of real wages and a component explained by the difference in PDV of search costs. Row 2 depicts the value of expression 8. Subrows decompose this difference in PDVs into a component explained by the difference in PDV of real output and a component explained by the difference in PDV of search costs.

Relative to the fully absorbing shock to trend inflation, the welfare costs of the shock to ε_p are more modest, as the inflation rate returns to its average value in steady-state. In turn, the real wage losses expected by workers, absent an increase in search effort, are lower relative to the case where the shock to the inflation rate is fully absorbing. This dampens the incentive to search, lowering costs borne by agents.

4.5 Costs of Price Instability

In this section, we analyze the efficiency properties of our equilibrium by comparing the outcomes under private search choice with those under a social planner who optimally chooses search effort to maximize total match surplus. We then use this framework to quantify how price instability amplifies these inefficiencies through multiple interacting channels.

The main purpose of this analysis is to provide insight into the inefficiencies of private search in a model where workers do not internalize the full social value of their labor. Since

of the Appendix.

wage setting only gives workers a portion of match surplus, this leads to two key margins of inefficiency. First, workers inefficiently search for outside offers to bid-up their wages within the same match—an action that a social planner would never take. Put differently, search policies depend on both wages *and* firm productivity, creating wage-dependent distortions. Second, workers do not internalize the full social benefit of mobility up the job ladder: their return is capped based on the social value of their current job.

The latter force is reflected in misallocation of workers to jobs, which drives down aggregate output. The former force is reflected in the social value of jobs being lower due to inefficiently high search policy choices. Therefore, in general, search policies are inefficient when search is selected privately. Workers at the top of the job ladder have incentives to search to secure rents, while workers at the bottom of the job ladder do not search enough.

4.5.1 Comparison of Match Values

To formalize this comparison, we first present the social value of a match under privately optimal search choice. When workers choose search effort to maximize their individual value $W(w, y, z)$, the resulting social match value is:

$$\begin{aligned} M^{priv}(w, y, z) = {} & Y(z, y) - C(s_e^*(w, y, z), z) + \beta\delta U(z') \\ & + \beta(1-\delta)(1-\lambda_g)(s_e^*(w, y, z) + \lambda_e) \int_{\underline{y}}^{q(w', y, z')} M(w', y, z')\, dF(x) \\ & + \beta(1-\delta)(1-\lambda_g)(s_e^*(w, y, z) + \lambda_e) \int_{q(w', y, z')}^{y} M(\phi^{\text{reneg}}, y, z')\, dF(x) \\ & + \beta(1-\delta)(1-\lambda_g)(s_e^*(w, y, z) + \lambda_e) \int_{y}^{\bar{y}} (M(\phi^{\text{poach}}, x, z') - \eta_e(x, z'))\, dF(x) \\ & + \beta(1-\delta)(1-\lambda_g)(1 - s_e^*(w, y, z) - \lambda_e) M(w', y, z') \\ & + \beta(1-\delta)\lambda_g \int_{\underline{y}}^{y} [M(w_u(x, z'), x, z') - \eta_u(x, z')]\, dF(x) \end{aligned} \tag{6}$$

where $s_e^*(w, y, z)$ is the privately optimal search effort chosen by workers to maximize their individual value function.

In contrast, the social planner chooses search effort to maximize the joint value of each match, leading to the socially optimal match value:

$$
\begin{aligned}
M^{sp}(y,z) = {} & Y(y,z) - C(s^{sp}(y,z),z) + \beta\delta U(z') \\
& + \beta(1-\delta)(1-\lambda_g)(s^{sp}(y,z)+\lambda_e)\int_{\underline{y}}^{\hat{y}(y)} M(y,z')\,dF(x) \\
& + \beta(1-\delta)(1-\lambda_g)(1-s^{sp}(y,z)-\lambda_e)M(y,z') \\
& + \beta(1-\delta)(1-\lambda_g)(s^{sp}(y,z)+\lambda_e)\int_{\hat{y}(y)}^{\bar{y}} (M(x,z')-\eta_e(x,z'))\,dF(x) \\
& + \beta(1-\delta)\lambda_g \int_{\underline{y}}^{y} [M(x,z')-\eta_u(x,z')]\,dF(x)
\end{aligned} \tag{7}
$$

The key difference is that the social planner's match value $M^{sp}(y,z)$ depends only on firm productivity y and aggregate productivity z, while the private social match value $M^{priv}(w,y,z)$ also depends on the real wage w in a contract. This wage dependence creates inefficiencies as workers with identical productivity but different wages will choose different search intensities.

4.5.2 Welfare Decomposition in Steady State

To quantify these inefficiencies, we compare aggregate welfare across the two equilibrium concepts. The equilibrium outcomes for search behavior can be represented by distinct distributions:

- Private search equilibrium: $G^{priv}(w,y,z)$ - the joint distribution over wages, firm productivity, and aggregate productivity
- Social planner's search equilibrium: $g^{sp}(y)$ - the marginal distribution over firm productivity
- Marginal distribution of y under private search: $g^{priv}(y)$ - obtained by integrating out wages from $G^{priv}(w,y,z)$

Using these distributions, we can compute average match values under each regime:

$$
\bar{M}^{sp} = \int M^{sp}(y,z) g^{sp}(y)\,dy \tag{8}
$$

$$
\bar{M}^{priv} = \int M^{priv}(w,y,z)\,dG^{priv} = \int \bar{M}^{priv}(y,z) g^{priv}(y)\,dy \tag{9}
$$

where $\bar{M}^{priv}(y,z) = \mathbb{E}[M^{priv}(w,y,z)|y,z]$ is the average match value at firm type y.

The welfare gap between social planner and private allocations can be decomposed into two distinct effects:

$$\begin{aligned}\Delta &\equiv \bar{M}^{sp} - \bar{M}^{priv} \\ &= \underbrace{\int M^{sp}(y,z)[g^{sp}(y) - g^{priv}(y)]\,dy}_{\text{(i) Allocative effect}} + \underbrace{\int [M^{sp}(y,z) - \bar{M}^{priv}(y,z)]g^{priv}(y)\,dy}_{\text{(ii) Within-employer policy wedge}} \end{aligned} \tag{10}$$

The **allocative effect** captures welfare differences arising from different distributions of workers across firm types. Under private search, workers may be inefficiently allocated across the productivity distribution due to distorted search incentives. This effect will be positive if the social planner achieves a more productive allocation of workers.

The **within-employer policy wedge** measures welfare losses from inefficient search policies conditional on the realized distribution of workers across firms. This effect captures the direct cost of workers making search decisions based on wage considerations rather than pure productivity considerations.

4.5.3 The Role of Price Instability

Price instability amplifies these inefficiencies through several channels. To understand these mechanisms, we compare welfare outcomes across two distinct economic environments. In the first, we consider our baseline model with price instability, where match values depend on inflation shocks: $M(w, y, \varepsilon)$. In the second, we examine a counterfactual economy without price instability, where match values are given by $M(w, y)$.

Each environment is characterized by its respective stochastic steady-state distribution of workers across wage-firm pairs. The economy with price instability generates a joint distribution $H(w, y, \varepsilon)$ over real wages w, firm productivity y, and inflation states ε. In contrast, the economy without price instability produces a distribution $G(w, y)$ over real wages and firm productivity alone.

Price instability distorts search effort choices through two primary mechanisms: its direct effect on real wages—operating similarly to a second-moment shock in stochastic steady state—and through its effect on expectations, given the persistence of inflation shocks. In our model, a more dispersed real wage distribution translates directly to a more dispersed search effort distribution, which in turn drives up search costs given the curvature in the search cost function. This effect is amplified through the persistence of inflation shocks.

The asymmetric role of shocks in our model further amplifies these costs. Workers at the top of the productivity distribution have different search incentives depending on whether they face inflationary or deflationary pressures, creating additional inefficiencies. Finally, these forces may engender a reallocation of workers up the job ladder in response to price instability.

4.5.4 Double Decomposition Framework

Our main quantitative exercise employs a "double decomposition" approach. We first compute the welfare wedge Δ between social planner and private search for different levels of inflation volatility:

- Δ_0: Welfare wedge in a benchmark economy with stable prices (no inflation volatility)
- Δ_k: Welfare wedge in an economy with inflation volatility level k

The additional welfare cost due to price instability is then:

$$\Delta_k^{\Delta} \equiv \Delta_k - \Delta_0 \tag{11}$$

This object captures how much price instability amplifies the existing inefficiencies in private search. To understand the channels through which this amplification occurs, we construct intermediate counterfactual distributions.

Let $\hat{H}(w, y) = H(w|y) \cdot G(y)$ be a hybrid distribution that combines the conditional wage distribution given firm type from the unstable economy, $H(w|y)$, with the marginal distribution of firm types from the stable economy, $G(y)$. This construction allows us to separate the effects of price instability on wage determination from its effects on worker allocation across firms.

Using these distributions, we can decompose the total welfare difference between the stable and unstable economies as:

$$\begin{aligned}
\Delta M &= \int M(w, y)\, dG(w, y) - \int M(w, y, \varepsilon)\, dH(w, y, \varepsilon) \\
&= \underbrace{\int M(w, y)\, dG(w, y) - \int M(w, y)\, d\hat{H}(w, y)}_{\text{Within-firm wage effect}} \\
&\quad + \underbrace{\int M(w, y)\, d\hat{H}(w, y) - \int M(w, y, \varepsilon)\, dH(w, y, \varepsilon)}_{\text{Composition and risk effects}}
\end{aligned} \tag{12}$$

Note that $\Delta_k^{\Delta} = \Delta M$ by construction, as both measure the additional welfare cost of price instability.

The first term captures the **within-firm wage effect**, measuring how price instability alters wage distributions conditional on firm productivity. By the law of iterated expectations, this effect can be written as:

$$\int M(w, y)\, dG(w, y) - \int M(w, y)\, d\hat{H}(w, y) = \int \Delta M(y) \cdot G(y)\, dy \tag{13}$$

where $\Delta M(y) = \int M(w, y)\, dG(w|y) - \int M(w, y)\, dH(w|y)$ represents the welfare difference attributable to wage distribution changes within firm type y.

The second term can be further subdivided to distinguish between compositional effects and direct risk effects. We define an additional intermediate distribution $\tilde{H}^0(w, y) = H(w|y) \cdot H(y)$, which uses the marginal distributions from the unstable economy for both wages conditional on firm type and firm types themselves, but abstracts from inflation risk. This allows us to write:

$$\begin{aligned} &\int M(w, y)\, d\hat{H}(w, y) - \int M(w, y, \varepsilon)\, dH(w, y, \varepsilon) \\ &= \underbrace{\left[\int M(w, y)\, d\hat{H}(w, y) - \int M(w, y)\, d\tilde{H}^0(w, y) \right]}_{\text{Composition effect}} \\ &\quad + \underbrace{\left[\int M(w, y)\, d\tilde{H}^0(w, y) - \int M(w, y, \varepsilon)\, dH(w, y, \varepsilon) \right]}_{\text{Risk effect}} \end{aligned} \tag{14}$$

Economic Interpretation of Each Component The **within-firm wage effect** isolates the contribution of welfare losses engendered by price instability on wage distribution within firms. It allows us to consider the extent to which price instability generates wage dispersion within matches, which in turn generates dispersion in search effort, driving down the social value of matches. This effect should be most pronounced at the top of the firm productivity distribution, where workers have the strongest incentives to engage in socially wasteful search to protect their wage levels.

The **composition effect** considers the extent to which there is a wedge in social value associated with reallocation of workers induced by search effort distortions. The composition effect measures how price instability changes the distribution of workers across firm types. In our model, this occurs because inflation shocks alter workers' search incentives, potentially leading to more or less job mobility up the firm productivity ladder.

The **risk effect** considers how the expectation of future wage erosion or wage gains alters search costs. The risk effect captures the direct impact of inflation uncertainty on match values, holding fixed the distributions of wages and firm types. This effect reflects how the presence of inflation risk per se affects the value of employment relationships, independent of any changes in actual wage or mobility outcomes.

4.5.5 Quantitative Results

We now present the quantitative results of our welfare decomposition analysis. Figure 15 decomposes the welfare gap between private search and the social planner's allocation across different levels of inflation volatility. Figure 16 provides a comprehensive decomposition of the welfare costs of price instability into its constituent channels.

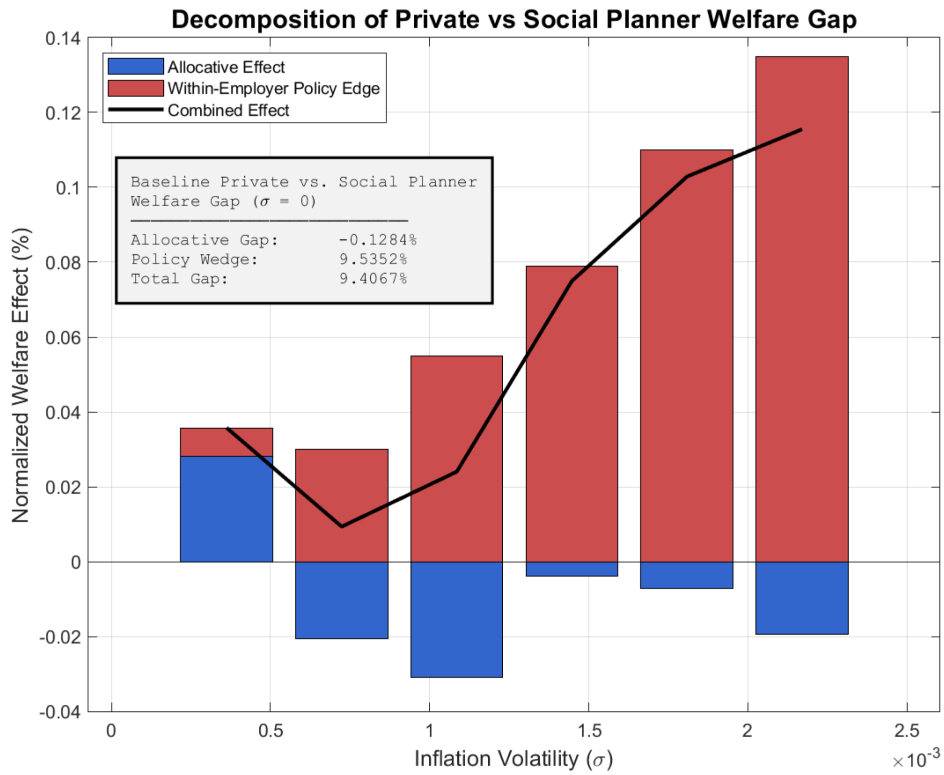


Figure 15: Decomposition of Private vs Social Planner Welfare Gap

Note: This figure decomposes the welfare gap between private search and social planner allocations as inflation volatility increases. The allocative effect measures welfare losses from differences in employment distributions across firm types, calculated as $\int M^{sp}(y,z)[g^{sp}(y) - g^{priv}(y)$. The within-employer policy wedge captures welfare losses, holding allocations of workers to firms fixed across the two economies, calculated as $\int[M^{sp}(y,z) - \bar{M}^{priv}(y,z)]g^{priv}(y)\,dy$. The combined effect represents the total welfare gap, equal to the sum of allocative and within-employer policy effects. All values are normalized by the social planner welfare level at $\sigma = 0$ and expressed as percentages. The baseline gap at $\sigma = 0$ shows an allocative gap of -0.1284% and a policy wedge of 9.5352%.

The decomposition in Figure 15 reveals several key findings. First, at baseline ($\sigma = 0$), the private search equilibrium exhibits a substantial welfare gap of approximately 9.41% relative to the social planner's allocation. This gap is almost entirely driven by the within-employer policy wedge (9.54%), while the allocative effect is slightly negative (-0.13%), indicating that the private search equilibrium actually achieves a marginally better distribution of workers across firms than the social planner.

As inflation volatility increases, the allocative effect evolves non-monotonically, becoming increasingly negative at moderate levels of inflation volatility before partially recovering at higher levels. This pattern suggests that price instability initially improves the allocation of workers across firms relative to the social planner, potentially because inflation shocks induce additional job-to-job transitions that move workers up the productivity ladder. The within-employer policy wedge, however, increases monotonically, increasing as inflation volatility rises, driving increased welfare losses.

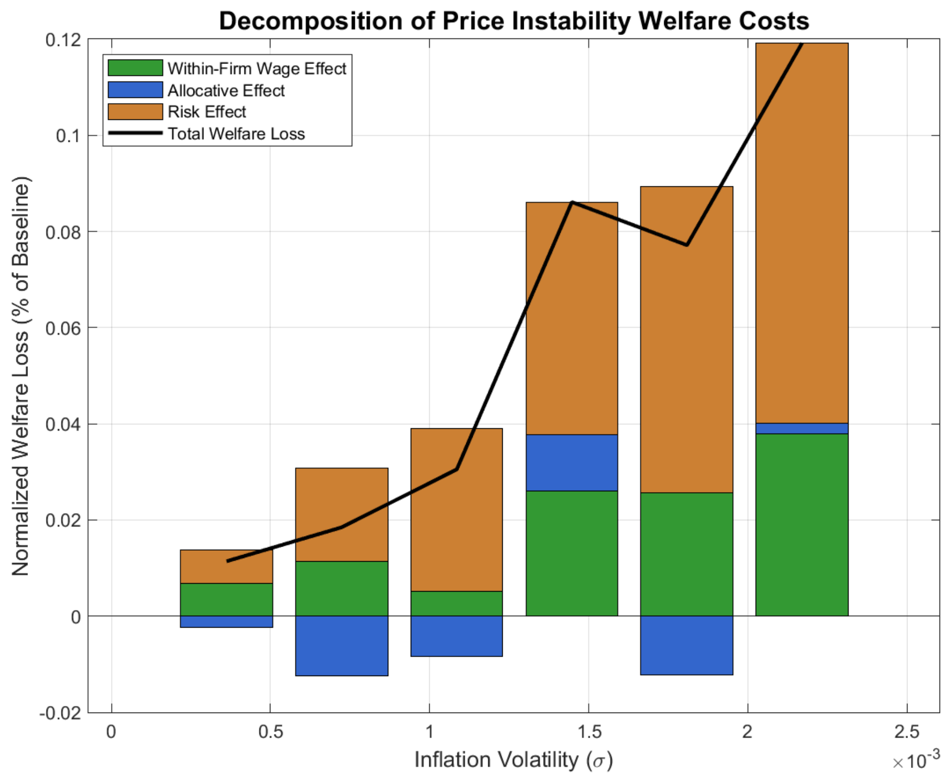


Figure 16: Decomposition of Price Instability Welfare Costs

Note: This figure decomposes the welfare costs of price instability relative to the stable price baseline. The within-firm wage effect captures welfare losses from increased wage dispersion within firm types due to inflation shocks, calculated using the counterfactual distribution $\hat{H}(w, y) = H(w|y) \cdot G(y)$. The allocative effect measures welfare changes from the reallocation of workers across firm types induced by price instability, isolated using intermediate counterfactual distributions. The risk effect represents the direct welfare cost of inflation uncertainty on match values, calculated as the residual after accounting for wage and allocation effects. Total welfare loss is the sum of all three effects, shown as the black line. All values are normalized by baseline social welfare and expressed as percentages of the stable-price economy welfare level.

Figure 16 provides our main decomposition of how price instability amplifies welfare losses in the private search equilibrium. The total welfare cost rises sharply with inflation volatility, reaching approximately 0.12% of baseline welfare at the highest volatility level ($\sigma = 0.00217$).

The decomposition reveals that the risk effect dominates at all levels of inflation volatility, accounting for roughly 60-70% of total welfare losses. This large risk effect reflects the direct cost of inflation uncertainty on match values—workers and firms face welfare losses simply from the possibility of future real wage erosion or gains, independent of any realized changes in wages or employment allocations.

The within-firm wage effect contributes approximately 20-30% of total welfare losses, increasing monotonically with inflation volatility. This effect captures how inflation shocks create wage dispersion within firm types, leading to heterogeneous and inefficient search responses. Workers with temporarily high real wages search excessively to protect their positions, while those with eroded wages may search too little relative to the social optimum.

The allocative effect exhibits interesting non-monotonic behavior, initially reducing welfare losses (negative values) before eventually contributing positively to total costs at higher volatility levels. This pattern suggests that moderate inflation volatility may actually improve the allocation of workers across firms by inducing additional job mobility, but this beneficial reallocation effect is dominated by distortions to search effort induced by within-firm wage dispersion and inflation expectations.

5 Conclusion

TBD.

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A Proof of Proposition 1

Proof. We will take a guess and verify approach. Start with the value of unemployment $U(z)$ evaluated at $U((1+g)z)$:

$$\begin{aligned} U((1+g)z) &= B((1+g)z) + \beta U\left((1+g)^2 z\right) \\ &= (1+g)B(z) + (1+g)U((1+g)z) \\ &= (1+g)U(z) \end{aligned}$$

Now let's look at the value of employment to the worker. First, using the implicit function for the renegotiated wage, multiplying both sides by $(1+g)$ we know that:

$$(1+g)W\Big(\phi^{\text{reneg}}(y_1,y_2,p,\varepsilon_p,z),y_1,p,\varepsilon_p,z\Big) = (1+g)W\Big(zy_2,y_2,p,\varepsilon_p,z\Big)$$

Using our guess for the scaling, the LHS is equivalent to:

$$W\Big((1+g)(1+g_p)\phi^{\text{reneg}}(y_1,y_2,p,\varepsilon,z),y_1,p(1+g_p),\varepsilon_p,z(1+g)\Big)$$

Defining

$$\phi^{\text{reneg}}(y_1,y_2,p(1+g_p),\varepsilon_p,z(1+g)) = (1+g)(1+g_p)\phi^{\text{reneg}}(y_1,y_2,p,\varepsilon,z) \tag{15}$$

implies:

$$W\Big(\phi^{\text{reneg}}(y_1,y_2,p(1+g_p),\varepsilon_p,z(1+g)),y_1,p(1+g_p),\varepsilon_p,z(1+g)\Big) = (1+g)W\Big(\phi^{\text{reneg}}(y_1,y_2,p,\varepsilon,z),y_1,p,\varepsilon_p,z\Big)$$

Using similar arguments,

$$W\Big(\phi^{\text{poach}}(y_2,y_1,p(1+g_p),\varepsilon_p,z(1+g)),y_2,p(1+g_p),\varepsilon_p,z(1+g)\Big) = (1+g)W\Big(\phi^{\text{poach}}(y_2,y_1,p,\varepsilon_p,z),y_2,p,\varepsilon_p,z\Big)$$

Finally, using the function which implicitly defines q:

$$W(w(1+g)(1+g_p),y_1,p(1+g_p),\varepsilon_p,z(1+g)) = (1+g)W(w,y_1,p,\varepsilon_p,z) = (1+g)W(zq,q,p,\varepsilon_p,z)$$

So q is unchanged. Turning to the value function and letting $\xi = (1-\delta)(s+\lambda_e), \xi' = (1-\delta)(1-s-\lambda_e)$, we have:

$$\begin{aligned} &W(w(1+g)(1+g_p),y,p(1+g_p),\varepsilon_p,z(1+g)) = \\ &\max_{s\in[0,\bar{s}_e]} (1+g)\frac{w}{p\varepsilon_p} - (1+g)c(s,z) + \beta(1+g)\delta U(z) \\ &+ \beta\mathbf{E}_{\varepsilon'_p}\xi \int^{q(w',y,p(1+g_p)^2,\varepsilon'_p,(1+g)^2 z)} W\left(w',y,(1+g_p)^2 p,\varepsilon'_p,(1+g)^2 z\right) dF(x) \\ &+ \beta\mathbf{E}_{\varepsilon'_p}\xi \int_{q\left(w',y,(1+g_p)^2 p,\varepsilon'_p,(1+g)^2 z\right)}^{y} W\left(\phi^{\text{reneg}}\left(y,x,(1+g_p)^2 p,\varepsilon'_p,(1+g)^2 z\right),y,(1+g_p)^2 p,\varepsilon'_p,(1+g)^2 z\right) dF(x) \\ &+ \beta\mathbf{E}_{\varepsilon'_p}\xi \int_y^{\bar{y}} W\left(\phi^{\text{poach}}\left(x,y,p',\varepsilon'_p,z'\right),x,(1+g_p)^2 p,\varepsilon'_p,(1+g)^2 z\right) dF(x) \\ &+ \beta\mathbf{E}_{\varepsilon'_p}\xi' W\left(w',y,(1+g_p)^2 p,\varepsilon'_p,(1+g)^2 z\right) \end{aligned}$$

where

$$w' = \begin{cases} \hat{w} : J\left(\hat{w}, y, p(1+g_p)^2, \varepsilon_p', z(1+g)^2\right) = 0, \text{if } \; J\left(w\left(1+g\right)^2\left(1+g_p\right)^2, y, p(1+g_p)^2, \varepsilon_p', z(1+g)^2\right) < 0 \\ \hat{w} : W\left(\hat{w}, y, p(1+g_p)^2, \varepsilon_p', z(1+g)^2\right) - U\left(z(1+g)^2\right) = 0, \\ \text{if } \quad W\left(w\left(1+g\right)\left(1+g_p\right), y, p(1+g_p)^2, \varepsilon_p', z(1+g)^2\right) - U\left(z(1+g)^2\right) < 0, \\ w\left(1+g\right)^2\left(1+g_p\right)^2, \text{else} \end{cases}$$

For the case where $w' = w(1+g)^2(1+g_p)^2$,

$$W\left(w', y, (1+g_p)^2 p, \varepsilon_p', (1+g)^2 z\right) = (1+g)W\left(w(1+g)(1+g_p), y, (1+g_p)p, \varepsilon_p', (1+g)z\right)$$

For the case where $w' = \hat{w}$ is defined implicitly using

$$J\left(\hat{w}, y, p(1+g)^2, \varepsilon_p', z(1+g)^2\right) = 0$$

the scaling conjecture implies:

$$\begin{aligned} &(1+g)J\left(\hat{w}\left(y, p(1+g_p), \varepsilon_p', z(1+g)\right), y, p(1+g), \varepsilon_p', z(1+g)\right) = \\ &\qquad J\left((1+g)(1+g_p)\hat{w}(y, p(1+g_p), \varepsilon_p', z(1+g)), y, p(1+g)^2, \varepsilon_p', z(1+g)^2\right) = 0 \end{aligned}$$

so that

$$\hat{w}\left(y, p(1+g)^2, \varepsilon_p', z(1+g)^2\right) = (1+g)(1+g_p)\hat{w}\left(y, p(1+g), \varepsilon_p', z(1+g)\right)$$

Similar arguments hold for the case where $w' = \hat{w}$ is defined implicitly using

$$W\left(\hat{w}, y, p(1+g)^2, \varepsilon_p', z(1+g)^2\right) - U(z(1+g)^2) = 0$$

Applying these results to the value function:

$$\begin{aligned} &W\left(w(1+g)(1+g_p), y, p(1+g_p), \varepsilon_p, z(1+g)\right) = \\ &\max_{s \in [0, \bar{s}_e]} (1+g)\frac{w}{p\varepsilon_p} - (1+g)c\left(s, z\right) + \beta(1+g)\delta U\left(z\right) \\ &+ \beta(1+g)\mathbf{E}_{\varepsilon_p'}\xi \int^{q(w', y, \varepsilon_p', p, z)} W\left(w', y, p(1+g_p), \varepsilon_p', z(1+g)\right) dF\left(x\right) \\ &+ \beta(1+g)\mathbf{E}_{\varepsilon_p'}\xi \int_{q\left(w', y, p, \varepsilon_p', z\right)}^{y} W\left(\phi^{\text{reneg}}\left(y, x, p(1+g_p), \varepsilon_p', z(1+g)\right), y, p, \varepsilon_p', z\right) dF\left(x\right) \\ &+ \beta(1+g)\mathbf{E}_{\varepsilon_p'}\xi \int_{y}^{\bar{y}} W\left(\phi^{\text{poach}}\left(x, y, p(1+g_p), \varepsilon_p', z(1+g)\right), x, p(1+g_p), \varepsilon_p', z(1+g)\right) dF\left(x\right) \\ &+ \beta(1+g)\mathbf{E}_{\varepsilon_p'}\xi' W\left(w', y, p(1+g_p), \varepsilon_p' z(1+g)\right) \end{aligned}$$

where now

$$w' = \begin{cases} \hat{w} : J\left(\hat{w}, y, p(1+g_p), \varepsilon_p', z(1+g)\right) = 0, \text{if } \; J\left(w\left(1+g\right)\left(1+g_p\right), y, p(1+g_p), \varepsilon_p', z(1+g)\right) < 0 \\ \hat{w} : W\left(\hat{w}, y, p(1+g_p), \varepsilon_p', z(1+g)\right) - U\left(z(1+g)\right) = 0, \\ \text{if } \quad W\left(w\left(1+g\right)\left(1+g_p\right), y, p(1+g_p), \varepsilon_p', z(1+g)\right) - U\left(z(1+g)\right) < 0, \\ w\left(1+g\right)\left(1+g_p\right), \text{else} \end{cases}$$

Therefore

$$W\left(w(1+g)(1+g_p),y,p(1+g_p),\varepsilon_p,z(1+g)\right)=(1+g)W\left(w,y,p,\varepsilon_p,z\right) \quad (16)$$

Similar arguments hold for the value of employment. ■

B Extended Model

For our quantitative exercise, we extend the model in three key dimensions. First, we allow the job destruction rate faced by workers to depend on firm productivity: $\delta(y)=\delta_0+\delta_1 y$. This embeds a "slippery" job ladder into our model. Second, we allow there to be exogenous job mobility shocks ("godfather shocks"), λ_g, under which a worker draws a job offer and either can accept the offer or receive the value of unemployment. Finally, we introduce moving costs η_e for EE transitions and η_u for UE transitions.

B.1 Value Functions

The value of employment to a worker is:

$$\begin{aligned}
W\left(w,y,p,p_\varepsilon,\varepsilon_p,z\right) &= \max_{s\in[0,\bar{s}_e]} \frac{w}{pp_\varepsilon}-c\left(s,z\right)+\beta(\delta(y)+(1-\delta(y))\lambda_g)U\left(z'\right)\\
&+\beta\mathbf{E}_{\varepsilon'_p}\left(1-\delta(y)\right)\left(1-\lambda_g\right)\left(s+\lambda_e\right)\int_{\underline{y}}^{q\left(w',y,p',p'_\varepsilon,\varepsilon'_p,z'\right)} W\left(w',y,p',p'_\varepsilon,\varepsilon'_p,z'\right)dF\left(x\right)\\
&+\beta\mathbf{E}_{\varepsilon'_p}\left(1-\delta(y)\right)\left(1-\lambda_g\right)\left(s+\lambda_e\right)\int_{q\left(w',y,p',p'_\varepsilon,\varepsilon'_p,z'\right)}^{\hat{y}(y)} W\left(\phi^{\text{reneg}}\left(y,x,p',p'_\varepsilon,\varepsilon'_p,z'\right),y,p',p'_\varepsilon,\varepsilon'_p,z'\right)dF\left(x\right)\\
&+\beta\mathbf{E}_{\varepsilon'_p}\left(1-\delta(y)\right)\left(1-\lambda_g\right)\left(s+\lambda_e\right)\int_{\hat{y}(y)}^{\bar{y}} W\left(\phi^{\text{poach}}\left(x,y,p',p'_\varepsilon,\varepsilon'_p,z'\right),x,p',p'_\varepsilon,\varepsilon'_p,z'\right)dF\left(x\right)\\
&+\beta\mathbf{E}_{\varepsilon'_p}\left(1-\delta(y)\right)\left(1-\lambda_g\right)\left(1-s-\lambda_e\right)W\left(w',y,p',p'_\varepsilon,\varepsilon'_p,z'\right)
\end{aligned}$$

where $p'=p\left(1+g_p\right)$, $p'_\varepsilon=p\left(1+\varepsilon'_p\right)$, and $z'=\left(1+g\right)z$.

The value of unemployment is:

$$\begin{aligned}
U(z) = \max_{s\in[0,\bar{s}_u]} \; & b(z)-c(s,z)+\beta U(z')\\
&+\beta s\mathbf{E}_{\varepsilon'_p}\int_{\underline{y}}^{\bar{y}}\max\left\{W(\phi_u,y,p',p'_\varepsilon,\varepsilon'_p,z'),U(z')\right\}dF(y)
\end{aligned} \quad (17)$$

The value to the firm is:

$$
\begin{aligned}
J\left(w, y, p, \varepsilon_p, z\right) &= Y\left(z, y\right) - \frac{w}{p p_{\varepsilon}} \\
&+ \beta(1-\delta(y))(1-\lambda_g)\lambda_e s_e^*\left(w, y, p, p_{\varepsilon}, \varepsilon_p, z\right) \mathbf{E}_{\varepsilon_p'} \int_{q\left(w', y, p', p_{\varepsilon}', \varepsilon_p', z'\right)}^{\hat{y}(y)} J\left(\phi_w^{\text{reneg}}\left(y, x, p', p_{\varepsilon}', \varepsilon_p', z'\right), y, p', p_{\varepsilon}', \varepsilon_p', z'\right) dF \\
&+ \beta(1-\delta(y))(1-\lambda_g)\lambda_e s_e^*\left(w, y, p, p_{\varepsilon}, \varepsilon_p, z\right) \mathbf{E}_{\varepsilon_p'} \left[\int^{q\left(w', y, p', p_{\varepsilon}', \varepsilon_p', z'\right)} dF\left(x\right)\right] J\left(w', y, p', p_{\varepsilon}', \varepsilon_p', z'\right) \\
&+ \beta\left(1-\delta(y)\right)\left(1-\lambda_g\right)\left[\left(1-\lambda_e s_e^*\left(w, p, p_{\varepsilon}, \varepsilon_p, z\right)\right)\right] \mathbf{E}_{\varepsilon_p'} J\left(w', y, p', p_{\varepsilon}', \varepsilon_p', z'\right)
\end{aligned}
$$

B.2 Moving Costs

Moving costs represent the utility loss workers experience when transitioning between jobs or from unemployment to employment. These costs are proportional to the surplus value of the new job:

$$\eta_e(y) = \eta_e \cdot \left[W(\hat{w}, y, p', p_{\varepsilon}', \varepsilon_p', z') - U(z')\right] \tag{18}$$

$$\eta_u(y) = \eta_u \cdot \left[W(\hat{w}, y, p', p_{\varepsilon}', \varepsilon_p', z') - U(z')\right] \tag{19}$$

where η_e^* and η_u^* are proportionality constants for job-to-job and unemployment-to-job transitions respectively, and $\hat{w}$ is the reservation wage such that the worker is indifferent between the job and unemployment.

Moving costs enter the model through the wage determination process. When a worker at firm y meets firm $x > \hat{y}(y)$ (poaching), the wage is set such that:

$$W\left(\phi^{\text{poach}}\left(x, y, p', p_{\varepsilon}', \varepsilon_p', z'\right), x, p', p_{\varepsilon}', \varepsilon_p', z'\right) = W\left(\hat{w}, y, p', p_{\varepsilon}', \varepsilon_p', z'\right) + \eta_e(x) \tag{20}$$

When a worker at firm y meets firm $x \in [q(w', y, p', p_{\varepsilon}', \varepsilon_p', z'), \hat{y}(y)]$ (renegotiation), the wage is set such that:

$$W\left(\phi^{\text{reneg}}\left(y, x, p', p_{\varepsilon}', \varepsilon_p', z'\right), y, p', p_{\varepsilon}', \varepsilon_p', z'\right) = W\left(\hat{w}, x, p', p_{\varepsilon}', \varepsilon_p', z'\right) - \eta_e(x) \tag{21}$$

These conditions ensure that after accounting for moving costs, workers are indifferent between their options. The moving cost structure implies that transitions to higher surplus jobs require larger wage premia to compensate workers for the utility loss of moving.

C Derivation of Social Planner's Problem

This section provides a detailed derivation of the social planner's value functions, showing how they relate to the individual optimization problem described in the main text.

C.1 From Individual to Social Optimization

The social planner chooses search effort to maximize the joint surplus of each match, defined as:

$$M(y,z) = W(w,y,p,p_\varepsilon,\varepsilon_p,z) + J(w,y,p,p_\varepsilon,\varepsilon_p,z) \tag{22}$$

Starting from the individual value functions in the main text, we substitute and simplify to derive the social planner's problem.

C.2 Step 1: Substitute Individual Value Functions

From the worker's value function:

$$\begin{aligned}
W(w,y,p,p_\varepsilon,\varepsilon_p,z) = \max_{s\in[0,\bar{s}_e]} & \frac{w}{p\cdot p_\varepsilon} - c(s,z) + \beta(\delta(y) + (1-\delta(y))\lambda_g)U(z') \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(s+\lambda_e) \\
& \times \int_{\underline{y}}^{q(w',y,p',p_\varepsilon',\varepsilon_p',z')} W(w',y,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(s+\lambda_e) \\
& \times \int_{q(w',y,p',p_\varepsilon',\varepsilon_p',z')}^{\hat{y}(y)} W(\phi^{reneg},y,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(s+\lambda_e) \\
& \times \int_{\hat{y}(y)}^{\bar{y}} W(\phi^{poach},x,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(1-s-\lambda_e)W(w',y,p',p_\varepsilon',\varepsilon_p',z')
\end{aligned} \tag{23}$$

From the firm's value function:

$$\begin{aligned}
J(w, y, p, p_\varepsilon, \varepsilon_p, z) = {} & Y(z, y) - \frac{w}{p \cdot p_\varepsilon} \\
& + \beta(1 - \delta(y))(1 - \lambda_g)(s + \lambda_e)\mathbf{E}_{\varepsilon_p'} \\
& \times \int_{q(w', y, p', p_\varepsilon', \varepsilon_p', z')}^{\hat{y}(y)} J(\phi^{reneg}, y, p', p_\varepsilon', \varepsilon_p', z') dF(x) \\
& + \beta(1 - \delta(y))(1 - \lambda_g)(s + \lambda_e)\mathbf{E}_{\varepsilon_p'} \\
& \times \left[\int_{\underline{y}}^{q(w', y, p', p_\varepsilon', \varepsilon_p', z')} dF(x) \right] J(w', y, p', p_\varepsilon', \varepsilon_p', z') \\
& + \beta(1 - \delta(y))(1 - \lambda_g)(1 - s - \lambda_e)\mathbf{E}_{\varepsilon_p'} J(w', y, p', p_\varepsilon', \varepsilon_p', z')
\end{aligned} \tag{24}$$

C.3 Step 2: Cancel Wage Terms

When we add the worker and firm value functions, the wage terms cancel:

$$\frac{w}{p \cdot p_\varepsilon} - \frac{w}{p \cdot p_\varepsilon} = 0 \tag{25}$$

C.4 Step 3: Use Key Observations About Match Values

Under the assumption that workers receive the full value of the match when hired (analogous to Cahuc, Postel-Vinay, Robin wage determination), we have:

For poaching (when $x > \hat{y}(y)$):

$$W(\phi^{poach}, x, p', p_\varepsilon', \varepsilon_p', z') = M(x, z') - \eta_e(x, z') \tag{26}$$

For renegotiation (when $q(w', y, p', p_\varepsilon', \varepsilon_p', z') < x \leq \hat{y}(y)$):

$$W(\phi^{reneg}, y, p', p_\varepsilon', \varepsilon_p', z') + J(\phi^{reneg}, y, p', p_\varepsilon', \varepsilon_p', z') = M(y, z') \tag{27}$$

For no offer case:

$$W(w', y, p', p_\varepsilon', \varepsilon_p', z') + J(w', y, p', p_\varepsilon', \varepsilon_p', z') = M(y, z') \tag{28}$$

C.5 Step 4: Substitute and Simplify

Using these relationships and combining terms:

$$
\begin{aligned}
M(y,z) = \max_{s\in[0,\bar{s}_e]} \{ & Y(z,y) - c(s,z) + \beta(\delta(y) + (1-\delta(y))\lambda_g)U(z') \\
& + \beta(1-\delta(y))(1-\lambda_g)(1-s-\lambda_e)\mathbf{E}_{\varepsilon_p'} M(y,z') \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \int_{\underline{y}}^{\hat{y}(y)} M(y,z')dF(x) \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \int_{\hat{y}(y)}^{\bar{y}} (M(x,z') - \eta_e(x,z'))dF(x) \Big\}
\end{aligned} \tag{29}
$$

C.6 Step 5: Derive Unemployment Value

Similarly, the value of unemployment under the social planner's allocation follows from the assumption that unemployed workers receive the full match value minus moving costs when hired:

$$
U(z) = \max_{s_u\in[0,\bar{s}_u]} \left\{ b - c(s_u,z) + \beta(\lambda_u + s_u) \int_{\underline{y}}^{\bar{y}} (M(x,z') - \eta_u(x,z'))dF(x) + \beta(1-\lambda_u - s_u)U(z') \right\} \tag{30}
$$

C.7 Step 6: First-Order Conditions

The socially optimal search effort satisfies:

$$
\begin{aligned}
c'(s^{sp}(y,z)) = & \mathbf{E}_{\varepsilon_p'}\beta(1-\delta(y))(1-\lambda_g) \int_{\underline{y}}^{\hat{y}(y)} M(y,z')dF(x) \\
& + \mathbf{E}_{\varepsilon_p'}\beta(1-\delta(y))(1-\lambda_g) \int_{\hat{y}(y)}^{\bar{y}} (M(x,z') - \eta_e(x,z'))dF(x) \\
& - \mathbf{E}_{\varepsilon_p'}\beta(1-\delta(y))(1-\lambda_g)M(y,z')
\end{aligned} \tag{31}
$$

For unemployed workers:

$$
c'(s_u^{sp}) = \beta \int_{\underline{y}}^{\bar{y}} (M(x,z') - \eta_u(x,z'))dF(x) - \beta U(z') \tag{32}
$$

This completes the derivation of the social planner's problem, showing how it emerges from the individual optimization framework while internalizing all externalities.

D Bilateral Efficiency

In this section, we derive the bilaterally efficient allocation where search effort is determined through a contract that maximizes the joint surplus of the worker-firm pair. This analysis helps us understand whether the search costs of inflation represent genuine welfare losses or merely reflect the inability of workers and firms to coordinate their responses to price shocks.

D.1 Bilateral Contracting Framework

Under bilateral efficiency, we assume that workers and firms can write complete contracts specifying search effort levels. The contracted search effort maximizes the joint value of the match, defined as:

$$M(y,z) = W(w,y,p,p_\varepsilon,\varepsilon_p,z) + J(w,y,p,p_\varepsilon,\varepsilon_p,z) \tag{33}$$

where both the worker's value function $W(\cdot)$ and the firm's value function $J(\cdot)$ now take the contracted search effort as given rather than optimizing over it individually.

D.2 Modified Value Functions

Under bilateral contracting, the worker's value function becomes:

$$\begin{aligned}
W(w,y,p,p_\varepsilon,\varepsilon_p,z) = {} & \frac{w}{p\cdot p_\varepsilon} - c(s,z) + \beta(\delta(y) + (1-\delta(y))\lambda_g)U(z') \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(s+\lambda_e) \\
& \times \int_{\underline{y}}^{q(w',y,p',p_\varepsilon',\varepsilon_p',z')} W(w',y,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(s+\lambda_e) \\
& \times \int_{q(w',y,p',p_\varepsilon',\varepsilon_p',z')}^{\hat{y}(y)} W(\phi^{reneg}(y,x,p',p_\varepsilon',\varepsilon_p',z'),y,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(s+\lambda_e) \\
& \times \int_{\hat{y}(y)}^{\bar{y}} W(\phi^{poach}(x,y,p',p_\varepsilon',\varepsilon_p',z'),x,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta\mathbf{E}_{\varepsilon_p'}(1-\delta(y))(1-\lambda_g)(1-s-\lambda_e)W(w',y,p',p_\varepsilon',\varepsilon_p',z')
\end{aligned} \tag{34}$$

Similarly, the firm's value function under bilateral contracting is:

$$
\begin{aligned}
J(w,y,p,p_\varepsilon,\varepsilon_p,z) = {} & Y(z,y) - \frac{w}{p\cdot p_\varepsilon} \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \\
& \times \int_{q(w',y,p',p_\varepsilon',\varepsilon_p',z')}^{\hat{y}(y)} J(\phi^{reneg}(y,x,p',p_\varepsilon',\varepsilon_p',z'),y,p',p_\varepsilon',\varepsilon_p',z')dF(x) \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \\
& \times \left[\int_{\underline{y}}^{q(w',y,p',p_\varepsilon',\varepsilon_p',z')} dF(x)\right] J(w',y,p',p_\varepsilon',\varepsilon_p',z') \\
& + \beta(1-\delta(y))(1-\lambda_g)(1-s-\lambda_e)\mathbf{E}_{\varepsilon_p'} J(w',y,p',p_\varepsilon',\varepsilon_p',z')
\end{aligned} \tag{35}
$$

D.3 Derivation of Joint Match Value

Combining the worker and firm value functions and noting that the wage terms $\frac{w}{p\cdot p_\varepsilon}$ cancel out, we obtain the joint match value:

$$
\begin{aligned}
M(y,z) = \max_{s\in[0,\bar{s}_e]} \{ & Y(z,y) - c(s,z) + \beta(\delta(y) + (1-\delta(y))\lambda_g)U(z') \\
& + \beta(1-\delta(y))(1-\lambda_g)(1-s-\lambda_e)\mathbf{E}_{\varepsilon_p'} M(y,z') \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \int_{\underline{y}}^{q(w',y,p',p_\varepsilon',\varepsilon_p',z')} M(y,z')dF(x) \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \int_{q(w',y,p',p_\varepsilon',\varepsilon_p',z')}^{\hat{y}(y)} M(y,z')dF(x) \\
& + \beta(1-\delta(y))(1-\lambda_g)(s+\lambda_e)\mathbf{E}_{\varepsilon_p'} \int_{\hat{y}(y)}^{\bar{y}} M(x,z')dF(x) \Big\}
\end{aligned} \tag{36}
$$

D.4 Bilaterally Efficient Search Choice

The key insight is that under bilateral efficiency, the search choice accounts for the complete impact on joint surplus. However, when moving costs $\eta_e(x,z) > 0$ are present, the bilaterally efficient search effort may be lower than what would maximize social welfare because renegotiation events reduce joint match value by the amount of the moving costs.

In the limit where moving costs are substantial, the bilaterally efficient search choice approaches:

$$
s^{bilateral}(y,z) = 0 \tag{37}
$$

This occurs because search effort only leads to either: (i) costly renegotiations that reduce match surplus by moving costs, or (ii) job transitions that may not improve total output sufficiently to offset the moving costs.

D.5 Comparison with Individual Optimization

The comparison between individual worker optimization, bilateral efficiency, and social planning reveals:

1. **Individual optimization**: Workers choose search to maximize their own value, ignoring impacts on firms
2. **Bilateral efficiency**: Worker-firm pairs choose search to maximize joint match value, but may not internalize externalities on other matches
3. **Social planning**: Search is chosen to maximize total economy-wide surplus, fully internalizing all externalities

E Model Solution

The model can be solved in the following steps:

1. For a given home production function $b(\Omega)$ and output functions $f(z, y)$ discount rate β, exogenous separation probability δ, the distribution of vacancies for all states $v(y, \Omega)$, the meeting rates for all states $\lambda(\Omega)$, exogenous search intensities for all states $s(\Omega)$, and stochastic transition matrices for z and p, $T(z, z')$ and $T_p(p, p')$, solve for the surplus function $S(y, \Omega)$ as the unique solution to Equation **??**. This implies a solution for $U(\Omega)$ and $M(y, \Omega)$.
2. Given some initial values for u_0 and $h_0(y)$, a sequence of stochastic productivity shocks $\{z_t\}_{t=0}^{T}$ and price level realizations $\{p_t\}_{t=0}^{T}$ imply a unique path for the unemployment rate, and the distribution of employed workers across firms:
$$\{u_t, h_t(y)\}_{t=0}^{T}$$
3. Given the path for the above objects, we can now turn to the dynamics of wages. To solve for wages, given some initial $\{z_0, p_0, u_0, h_0(y)\}$:[22]
   (a) Construct a grid of wage outcomes, $w^j(y, p, y_l, p_l, z, z_l)$, where j refers to the iteration of the solution algorithm.

[22]The easiest is to begin with everyone in unemployment, so that the surplus shares are irrelevant.

(b) Guess an initial value function for $W(y, p, y_l, p_l, z, z_l)$.

(c) Construct $\sigma(y, p, y_l, p_l, z, z_l)$, the implied share of surplus the nominal wage $w^j(\cdot)$ generates for the worker.

(d) Construct $c_e^*(y, p, y_l, p_l, z, z_l)$, the cutoff search cost value.

(e) Iterate on $W(y, p, y_l, p_l, z, z_l)$ using Equation **??** until convergence.

(f) Given the updated value $W(y, p, y_l, p_l, z, z_l)$, we can solve for wages for those coming out of unemployment which must satisfy $W(y, p, \emptyset, p, z, z) - U(p, z) = 0 \quad \forall p, z$.[23] We can also solve for wages for any worker transitioning from one firm y to another (equal or higher surplus firm) y' when the state is Ω as $W(y', p, y, p, z, z) - U(z) = S(y, z) \quad \forall p, z$. As $W(y', p, y, p, z, z) = \frac{w(y', p, y, p, z, z)}{p} + W_{cont}(y', p, y, p, z, z)$, $w(y', p, y, p, z, z) = p * (U(z) + S(y) - W_{cont}(y', p, y, p, z, z))$

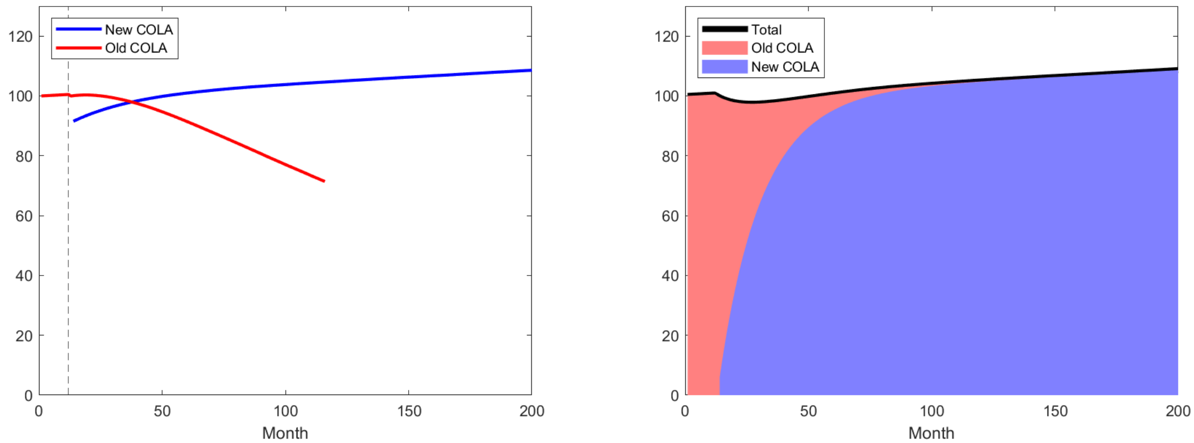
(g) In cases where $p \neq p_l, z \neq z_l$, $w(y', p, y, p_l, z, z_l) = p_l * (U(z_l) + S(y, z_l) - W_{cont}(y', p, y, p, z, z))$ with the restriction that $\max_{y, y_l, z_l, p_l}(w(y', p, y, p_l, z, z_l)) = p * (U(z) + S(y, z) - W_{cont}(y', p, y, p, z, z)))$ and $\min_{y, y_l, z_l, p_l}(w(y', p, y, p_l, z, z_l) = p * (U(z) - W_{cont}(y', p, y, p, z, z)))$. This says that aggregate shocks z, p may force a renegotiation of the wage contract in the following two cases: (1) absent renegotiation, the contracted wage would result in the employer laying off the worker (2) absent renegotiation, the contracted wage would result in the worker quitting into unemployment.

(h) Given this new wage grid, return to (c) and repeat steps (c)-(d) until convergence.

[23] The $\emptyset$ notation refers to the state of unemployment.

F Additional Figures

F.1 Movement in Trend Inflation

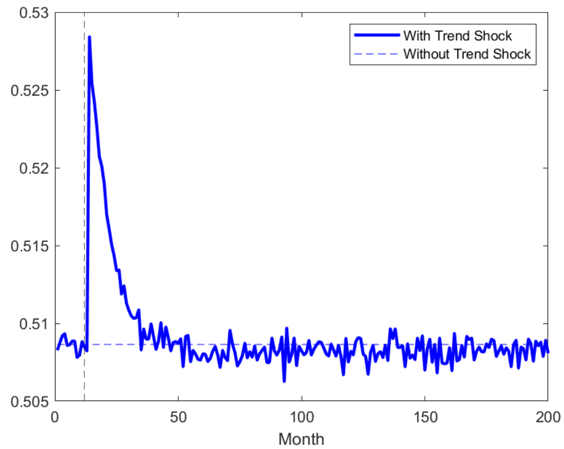


(a) Average real wages by contract type

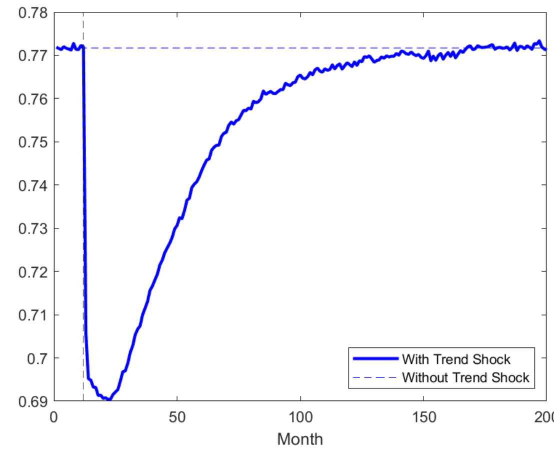
(b) Real wages decomposition

Figure 17: Real Wages and COLAs in Wage Contracts

Note: Figure 17a plots average real wages by wage contract types, indexed relative to the overall average real wage. Figure 17b decomposes the average real wage series into a part explained by the wages of workers with a COLA negotiated at g_p^{low} and a part explained by the wages of workers with a COLA negotiated at g_p^{high}



(a) Acceptance ratio



(b) EE share of all contacts

Figure 18: Mechanisms of adjustment

Note: Figure 18a plots the $\frac{EE}{UE}$ ratio - described as the acceptance ratio in Moscarini and Postel-Vinay (2017). Figure 18b plots the share of contacts, defined as renegotiation or job-to-job transition events, that are job-to-job transitions.

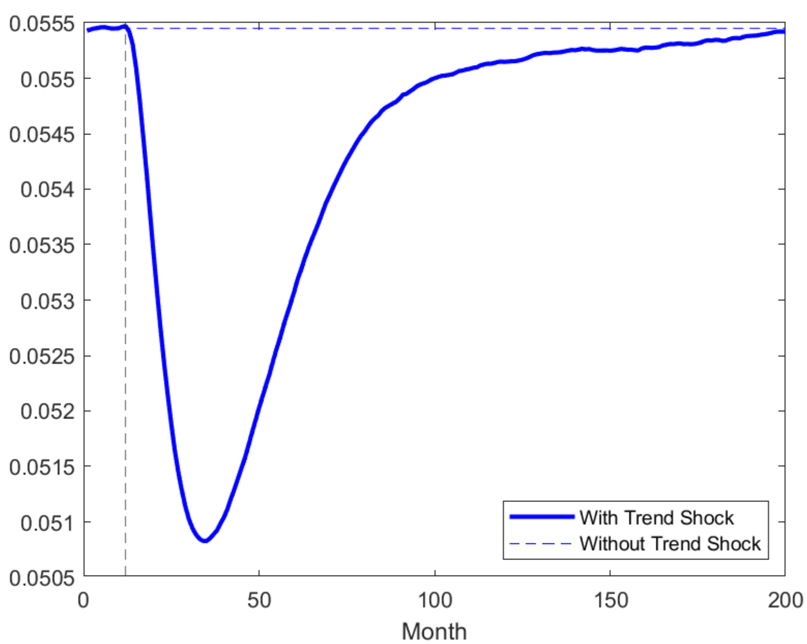
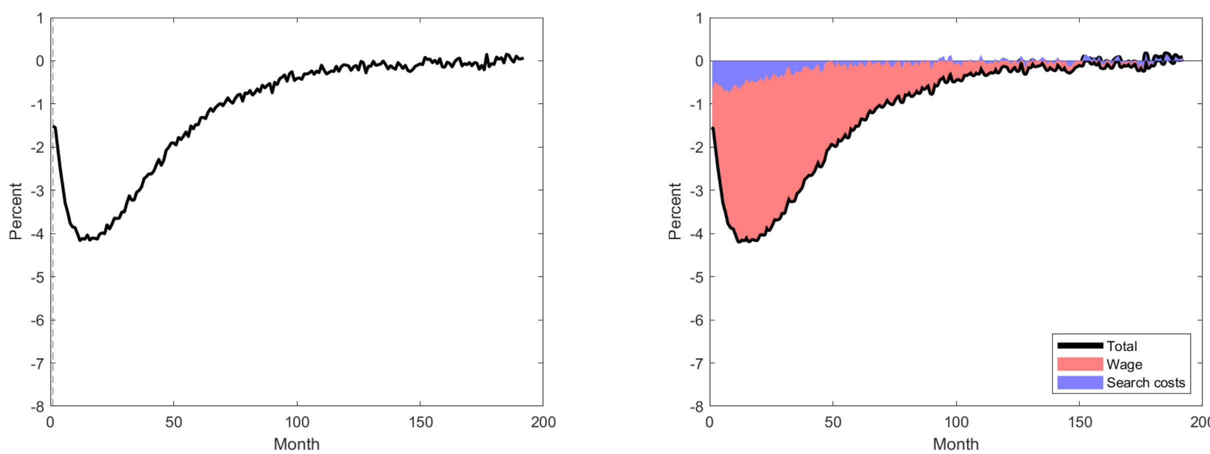


Figure 19: Variance of log wages

Note: Figure 19 displays the evolution of $Var(ln(w))$ following the trend inflation shock.

F.2 Welfare Costs of a Persistent Inflationary Shock

In Figure 20a, we display this measure of the average welfare losses experienced by workers in the shocked economy per period, where the first month plotted is the first month of the shock [24]. In Figure 20a, we may observe that the per-period average welfare losses experienced by workers are felt immediately, even as real wages are almost unchanged in the first period: this occurs because search effort immediately increases in response to the inflationary shock, as workers expect inflation to remain elevated in the future. The per-period welfare losses borne by workers increase substantially over time as inflation runs hot, reaching their maximum at around 4 percent, as the real wages of workers continue to erode over time. The per-period losses of workers only recover slowly, even as the inflation rate returns to baseline 5 years later, remaining significantly negative 10 years after the shock. The main takeaway is that, from the perspective of a worker, our model suggests that a large inflationary shock is very costly: even if workers are able to anticipate high inflation in the future, increasing search effort accordingly, they pay a substantial cost for doing so, and their real wages fall substantially.



(a) Worker welfare losses

(b) Decomposition of worker welfare losses

Figure 20: Per-period Welfare Costs of an Inflationary Shock: Workers

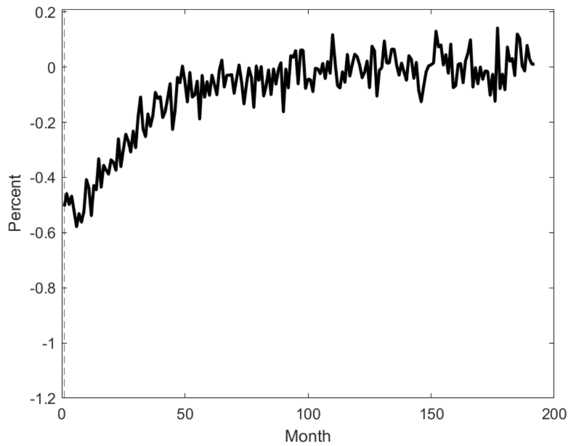
Note: Figure 20a plots the difference between the average real wage net of search costs in the shocked economy and the baseline economy in percentage terms. Figure 20b decomposes the sources of this percentage difference into the amount explained by differences in real wages, represented by the red shaded area, and the amount explained by differences in search costs, represented by the blue shaded area.

Below, we display this measure of the aggregate welfare losses experienced by agents in the

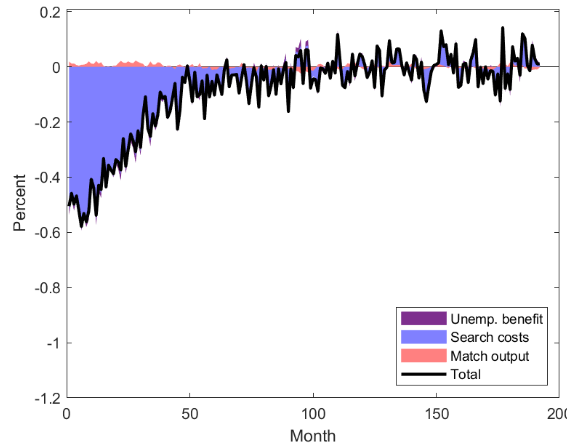
[24] To consider the welfare costs induced by the large inflationary shock, we perform the same calculations as in the previous section.

shocked economy per period, where the first month plotted is the first month of the shock. In Figure 21a, we may observe that the per-period average welfare losses experienced by agents are felt immediately, as the anticipation channel induces agents to ramp up their search activist, causing a 0.5 percent reduction in real value received by agents in the shocked economy relative to the baseline economy. As the inflation rate declines, per-period welfare losses gradually diminish, becoming negligible baseline around 8 years later. The main takeaway is that a large, unanticipated and persistent inflationary shock is quite costly to the economy as a whole, with the search costs of inflation dominating the reallocation channel.

As shown in Figure 21b, which decomposes the sources of the per-period losses faced by agents into a portion explained by (i) differences in real average match output (ii) differences in average search costs (iii) differences in unemployment benefits, explained by differences in employment rates, most of the per-period welfare losses are, indeed, experienced by agents in the form of increased search costs as seen in the blue shaded region. This highlights the fact that the per-period search costs of inflation are quite large, accounting for 1 percent of value. The reallocation of workers towards better firms that occurs due to increased search effort only results in a small increase in real output. This takes place due to two distinct channels. First, as workers are expending more search effort, they move up the job ladder, so average productivity of matches increases. Second, as job destruction rates are decreasing in firm productivity, the employment rate increases slightly, further increasing output. The second channel implies that the role of home production in the shocked economy is smaller than in the baseline economy. As the ratio of the flow value of unemployment to average match output is quite high, this results in only small output increases. Therefore, on net, the reallocative role of the inflationary shock is trivial in comparison to the role of the search costs of inflation.



(a) Aggregate welfare losses



(b) Decomposition of aggregate welfare losses

Figure 21: Aggregate Per-period Welfare Costs of an Inflationary Shock

Note: Figure 21a plots the difference between the average output per worker net of search costs in the shocked economy and the baseline economy in percentage terms. Figure 21b decomposes the sources of this percentage difference into the amount explained by differences in real output, represented by the red shaded area, the amount explained by differences in search costs, represented by the blue shaded area, and the amount explained by differences in home production, represented by the purple shaded area.

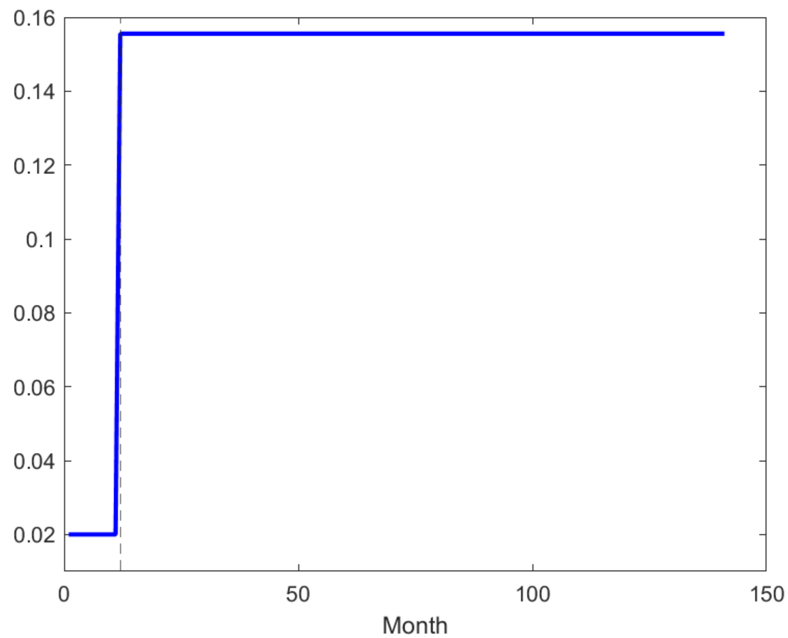
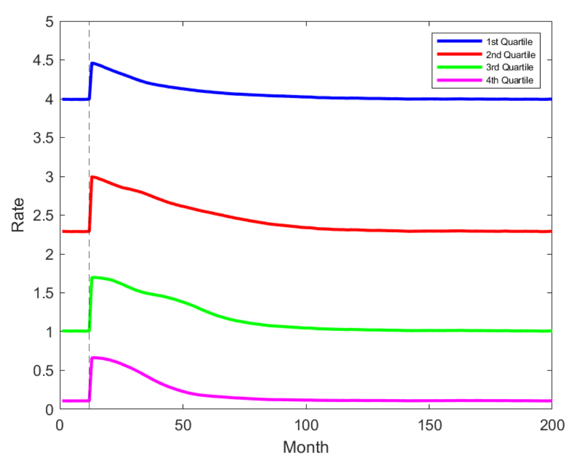


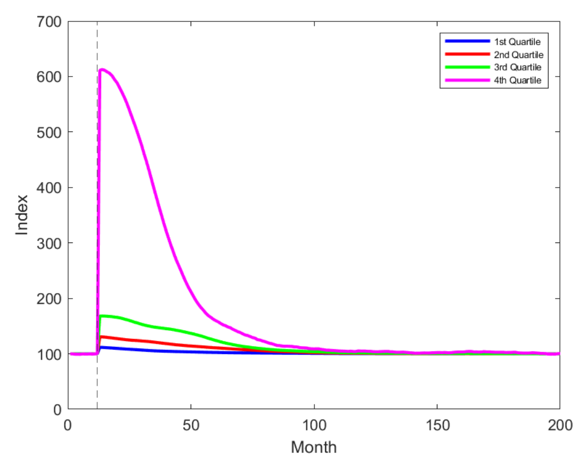
Figure 22: Expected Inflation Rate

Note: Figure 22 displays the evolution of expected inflation, overlayed with the realized inflation rate.

F.2.1 Additional Figures on Heterogeneous Adjustment



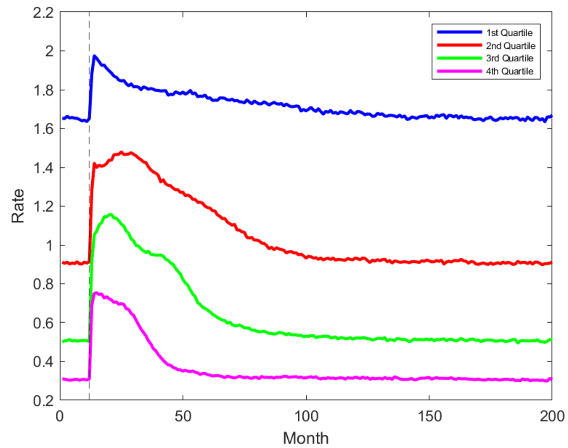
(a) Search Effort Levels by Quartile



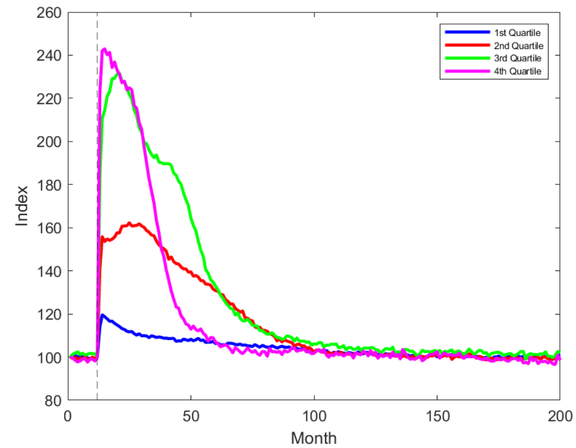
(b) Search Effort Indexed to Baseline

Figure 23: Search Effort Response by Wage Quartile

Note: Panel (a) shows the level of search effort by wage quartile. Panel (b) shows search effort indexed to 100 at baseline, highlighting the proportional increase following the shock.



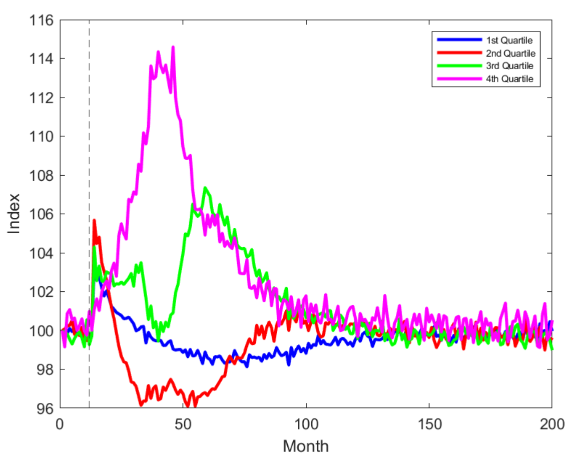
(a) Renegotiation Rate Levels



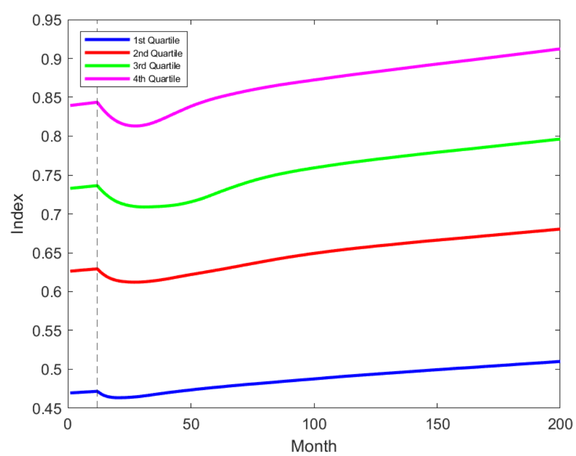
(b) Renegotiation Rates Indexed

Figure 24: Wage Renegotiation Patterns by Quartile

Note: Panel (a) displays wage renegotiation rates by quartile. Panel (b) shows these rates indexed to baseline. Lower wage workers experience dramatically higher renegotiation rates.



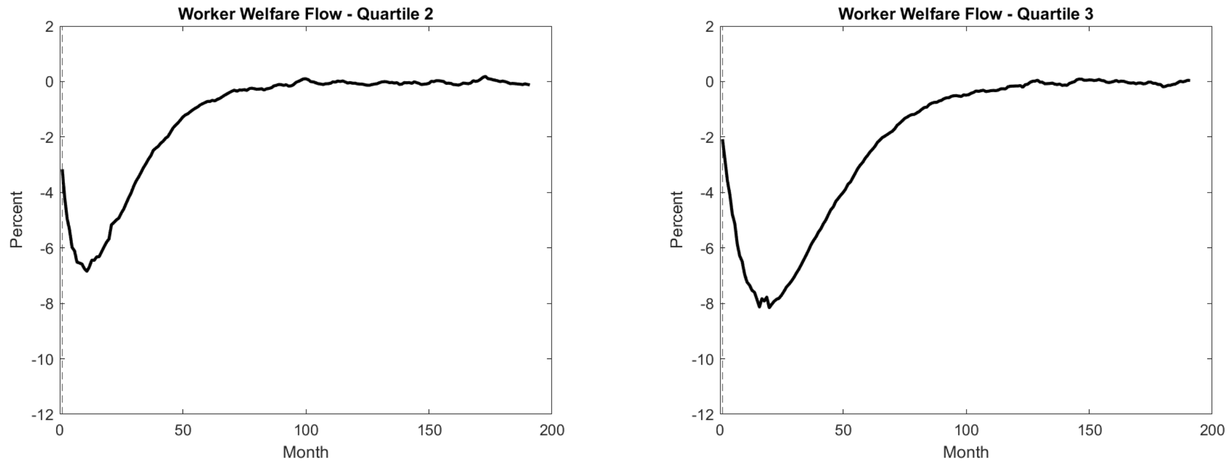
(a) Job Switching Rates



(b) Real Wage Levels

Figure 25: Additional Labor Market Dynamics by Quartile

Note: Panel (a) shows job-to-job transition rates indexed to baseline. Panel (b) displays real wage levels by quartile, demonstrating persistent level differences.

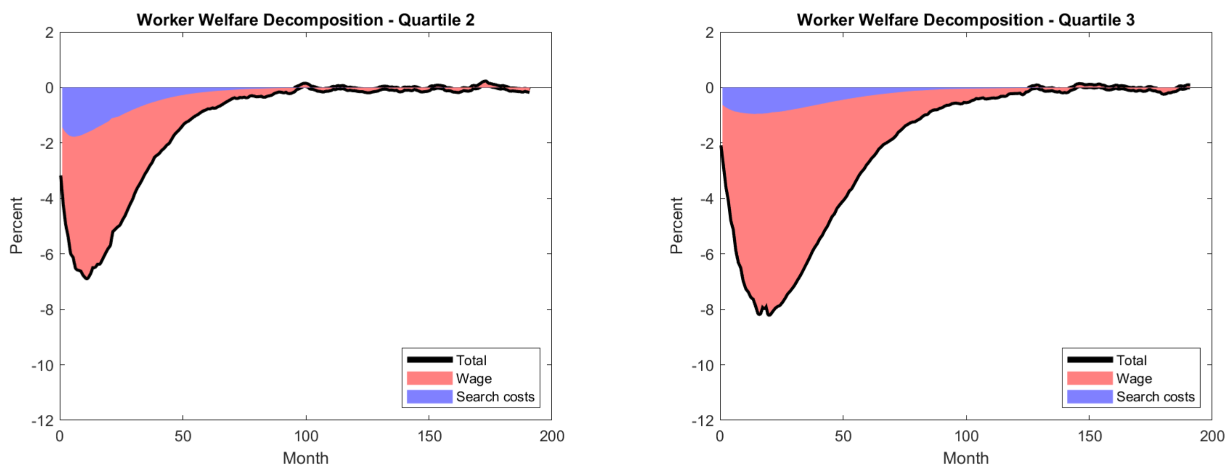


(a) Second Quartile

(b) Third Quartile

Figure 26: Worker Welfare Losses: Middle Quartiles

Note: Per-period welfare losses for workers in the second and third wage quartiles, showing intermediate patterns between the extremes.



(a) Second Quartile

(b) Third Quartile

Figure 27: Welfare Loss Decomposition: Middle Quartiles

Note: Decomposition of welfare losses for middle wage quartiles, showing the transition from search-cost-dominated losses (Q2) to wage-loss-dominated losses (Q3).

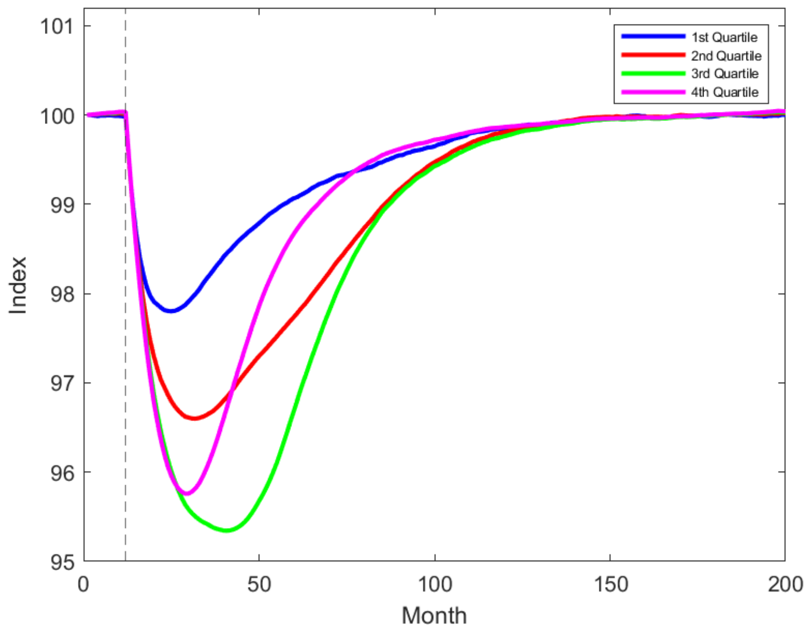


Figure 28: Detrended Real Wages by Quartile

Note: Real wages by quartile after removing the deterministic growth trend, highlighting the pure effect of the inflation shock on relative wages.